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Economic study of plutonium recycle in a thermal reactor

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by

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INTRODUCTION

During the next decade the existing power reactors will be in operation as well as many new power reactors which will be brought on line. The fuel needs for these reactors as well as the amount of discharged fuel will continue to grow. In view of the fact that plutonium isotopes build up during power reactor operation as a result of an initial neutron capture in uranium-238, there will be large amounts of plutonium available in the near future. With a large supply of plutonium available, it is expected that the market price will fall until plutonium recycle in thermal reactors becomes economically attractive. Later, with the introduction of fast breeder reactors utilizing plutonium as fuel, the plutonium market price will probably rise, and it may no longer be economical to recycle plutonium in thermal reactors until the breeder reactors produce an over supply of plutonium.

There are four important plutonium isotopes that build up in a thermal reactor as a result of an initial capture in uranium-238. Of these there are two isotopes, plutonium-239 and -241, which are fissile. Hence the performance and economics of a uranium-plutonium fuel mixture in the reactor will depend upon the isotopic composition of the plutonium, the total amount of plutonium in the fuel mixture, and the enrichment selected for uranium-235.

Since large amounts of money are invested in the nuclear

fuel cycle, small differences in the performance characteristics of fuel mixtures may result in considerable savings. A study to determine how different recycle fuel mixtures may affect the fuel cycle costs is therefore well justified. In this study several different parameters affecting the fuel cycle costs are varied. The results are helpful in making certain general conclusions about possible management decisions concerning the recycle of plutonium in thermal reactors.

REVIEW OF LITERATURE

There have been several economic studies concerned with plutonium recycle in thermal reactors. Examples of such studies include estimating fabrication penalties associated with particular reactor designs, developing scattering kernels applicable to plutonium-uranium-water lattices, and offering specific expressions for determining the value of plutonium as a thermal reactor fuel. None of the references reviewed, however, gave any appreciable data which might be used to evaluate which isotopic mixtures of recycle fuels would be the most economically desirable.

In obtaining general information for this study extensive use was made of the texts by Glasstone and Sesonske [5] and Lamarsh [11]. Benedict and Pigford have treated the buildup of isotopes with plutonium recycle in a simple manner that can be easily generalized [2].

Cross section data were obtained from United States Atomic Energy Commission Report BNL-325 [6,8]. References [11,20,23] were used to consider doppler broadening effects. The neutron spectrum was obtained by using data from a paper from the proceedings of a symposium on the use of plutonium as a reactor fuel [4]. Spectra for a plutonium-uranium-water lattice core were given for different traces of plutonium present in the fuel. An article presenting the spectral results of experiments utilizing different isotopic

solutions of plutonium nitrate was also used [15].

A graph giving the isotopic composition of commercial plutonium as a function of fuel exposure in a thermal reactor was found to be quite useful in estimating what recycle fuels may be available in the near future [25]. The United States Atomic Energy Commission has published information of a typical pressurized water reactor design in WASH-1082 [10]. This reference design was used extensively as a model reactor in this study.

Helpful information for calculating fuel cycle costs was found in several sources [9,10,12,13,19,20,22,26]. Recent cost estimates for different components of the nuclear fuel cycle were taken from The Nuclear Industry 1971 [26].

In addition to references giving information on basic core physics and fuel cycle costs, it was basic to this study to review literature specifically concerned with the physics and economic aspects of using plutonium in thermal reactors. Dawson has given an excellent paper concerned with in-core physics [22]. Nuclear Technology has devoted an entire issue to plutonium recycle problems of current interest [18]. Other important papers and articles were concerned with the fabrication penalty, supply and demand of plutonium, the value of plutonium as a thermal reactor fuel, and philosophies of private enterprise toward plutonium recycle [7,14, 16,17,21,24].

ECONOMIC ANALYSIS OF THE PLUTONIUM RECYCLE FUEL CYCLE

An economic study of plutonium recycle in a thermal reactor involves summing the cost components of the nuclear fuel cycle for the reactor, applying a technique to determine the interest charges for the use of capital during the fuel cycle time period, and conducting a physics study to determine the buildup of important isotopes in the reactor as well as to assure criticality of the reactor for the desired time period.

The Plutonium Recycle Fuel Cycle for a Thermal Reactor

The basic components of the nuclear fuel cycle with plutonium recycle in a thermal reactor are illustrated in the flow diagram in Figure 1. In the present discussion interest charges for the use of capital will be ignored.

Purchase of U_3O_8 and Recycle Fuels

The starting point of the nuclear fuel cycle is to purchase U_3O_8 obtained from uranium bearing ores, and/or recycle fuels. The mass of U_3O_8 which must be purchased is dependent on the feed supply required during the enrichment process, the magnitude of material losses in the conversion and fabrication processes, and the extent of recycle. The market price of U_3O_8 depends on the supply and demand. In 1971 the supply exceeded the demand, and the market price was a little less than $7 per pound [26]. Future projections indicate that this price will increase moderately [26].

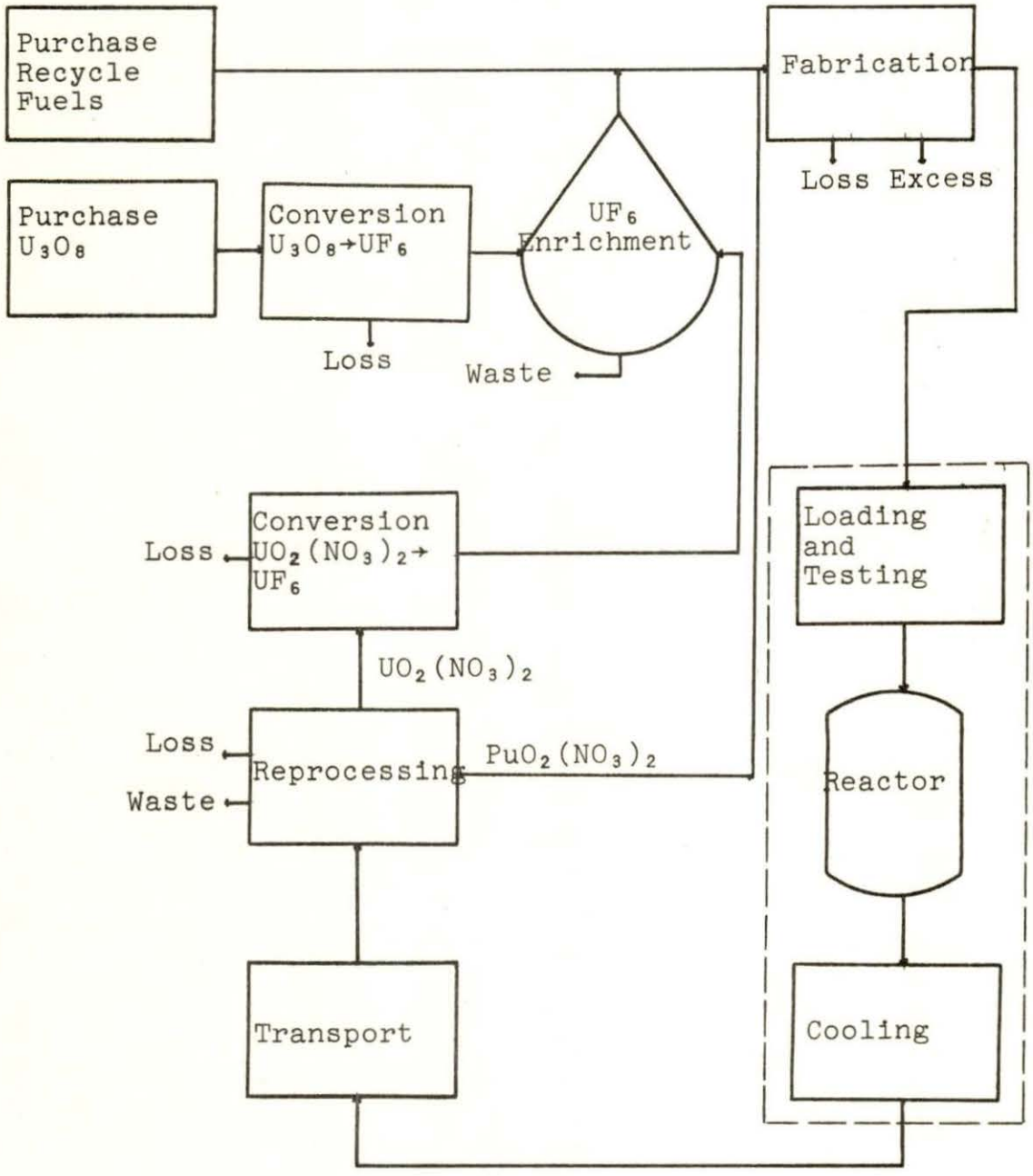

Figure 1. The nuclear recycle fuel cycle

The purchase price of recycle uranium will be dependent on the cost of producing uranium of an equal enrichment less a penalty for the presence of uranium-236, which is a poison in the reactor and complicates the enrichment process. However, its presence in a reactor can lead to the production of plutonium-238, and the cost penalty will depend on the market price of this isotope.

The market price of plutonium will depend upon its supply and demand as a research and development material, weapons material, light water reactor fuel, and perhaps in the future as a fast breeder reactor fuel. One projection states that plutonium recycle values will likely range from \$3 to 7/gram of fissile content until 1976 and then increase to a range of \$7 to 9/gram in the early 1980's [14]. This will depend on the length of time before the fast breeder reactor is commercially accepted.

In determining the value of plutonium as a thermal reactor fuel a penalty for the presence of plutonium-242 and higher costs of fabrication may be applied. The AEC has published an expression for the plutonium value as [10]:

$$\text{Plutonium Value (\$/gm fissile)} = gU_{93}(1-0.6P_{42})(1-W\Delta F) \quad (1)$$

where

P_{42} = grams of plutonium-242 per gram of fissile material

U_{93} = value of uranium enriched to 93 per cent

$W\Delta F$ = the fabrication "discount factor"

The value of g for light water reactors is given as 0.8 to 1.0.

Another study considered the value of plutonium as equal to the sum of the value of each individual isotope present [24]. For the recycle cases investigated it was found that the approximate value of plutonium could be represented by this method as:

$$\text{Plutonium value (\$/Kg)} = 10657 \cdot f_{49} + 2619 \cdot f_{40} + 11160 \cdot f_{41} - 10486 \cdot f_{42} \qquad (2)$$

where f_i is the fraction of the ith isotope in the mixture. It may be observed that again there is a penalty for the presence of plutonium-242. A similar penalty does not occur for plutonium-240 since neutron capture in this isotope produces fissile plutonium-241.

Conversion of U_3O_8 to UF_6

The enrichment of uranium-235 by the gaseous diffusion process, whereby gases having different molecular weights diffuse through a porous barrier at different rates, necessitates that the U_3O_8 be converted to UF_6. In this study the cost of conversion will also include sampling of the ore concentrate and transportation costs. During the chemical processes of converting the ore to UF_6 there are some losses of material. The cost of these losses are included as extra mass of U_3O_8 which must be purchased.

At the present time the United States commercial capability to convert ore concentrates of UF_6 consists of two plants. The base charge for converting ore concentrates to

UF_6 runs very close to $1.25 per pound of uranium [26].

Enrichment of Uranium-235

The enrichment of uranium-235 is presently available only from the AEC through toll enrichment in gaseous diffusion plants. A derivation for the total flow rate in an ideal cascade gaseous diffusion plant is given in Reference [2]. It is found to be the product of a factor indicating the relative ease or difficulty of the separation, and a factor which is proportional to the throughput denoted as the separative duty, and written as:

$$S = W \cdot \phi(x_W) + P \cdot \phi(x_p) - F \cdot \phi(x_f) \qquad (3)$$

where

F, P, W = mass of feed, product, and waste material, respectively

x_f, x_p, x_W = atom fraction of uranium-235 in the feed, product, and waste material, respectively

and $\phi(x)$ is the separation potential given by:

$$\phi(x) = (2x-1) \cdot \ln[x/(1-x)]$$

In a gaseous diffusion plant built as an ideal cascade of stages, the total flow rate, the total pump capacity, the total power demand, and the total barrier area are all proportional to the separative duty, and hence the separation.

The total charge of enriched material is then:

$$C_E = F \cdot C_F + S \cdot C_S \qquad (4)$$

where

F = mass of feed material

C_F = cost of the feed material

S = separative work (duty)

C_S = cost of enrichment per separative work unit

As the tails atom fraction approaches the feed atom fraction, the cost of the feed increases. As the tails atom fraction approaches zero, the separative cost increases. There exists, therefore, a tails composition that will give a minimum cost, found by setting:

$$\frac{\partial C_E}{\partial x_w} = 0 \qquad (5)$$

Hence, by specifying the tails composition, a utility should be able to obtain the minimum enrichment costs. In practice, however, the AEC specifies the tails composition and supplies the customer with an amount of enriched product and waste equal to the amount supplied.

The diffusion plants operate at a tails assay of 0.003, but the AEC charges the customer for enriching services as if the plants operated at a 0.002 tails assay [16 ,20]. The difference in the amount of feed required is drawn from the AEC stockpile of 50,000 tons of natural uranium. The purpose is to delay the construction of a new diffusion plant and to prevent the flooding of the uranium market with surplus uranium.

Fabrication of the Fuel Elements

Fabrication costs include the costs of hardware, pelletizing, shaping, and machining of the fuel material, the

fuel cladding material, assembly of the fuel elements, and quality control costs. Fabrication costs for uranium-plutonium mixed oxide fuels are expected to be greater than uranium oxide fuel fabrication costs. This is because special precautions must be taken during plutonium fabrication and handling due to its toxic and radioactive characteristics. Also small diameter fuel rods may be necessary since the optimum water-to-fuel ratio of plutonium is higher than for uranium.

It is expected that the fabrication of mixed oxide fuels will be 20 to 100 percent higher than uranium oxide fuels. The AEC gives an estimate of the fabrication costs of uranium oxide fuels for 1971 as $70/KGU [26].

During the fabrication process there will be some loss of material and some excess material. The cost of losses is included in the extra mass of fuel that need be purchased and enriched.

Credit for Excess Material

The excess material from fabrication is credited to the customer. The amount of this credit will depend upon the isotopic composition of the fuel.

Operation and Cooling

During the operation of the reactor, the fuel is depreciated as fissile material is used to generate electricity. There are a number of methods which may be used to calculate the depreciation of the fuel. In addition,

interest charges may accumulate during this time. These are discussed later.

Transport of Spent Fuel

Transportation of the spent fuel is a significant cost since special casks and transportation arrangements are necessary to contain, shield, and cool the radioactive material. Transport costs are about $5/KGH discharged [26].

Reprocessing of the Spent Fuel

Reprocessing of the nuclear fuel elements includes removal of the cladding material, separation of the fission products from uranium and plutonium, and reconversion of the uranium to UF_6. The AEC gives an estimate of the reprocessing costs in 1971 as $35/KGH discharged [26].

Credit for Discharged Uranium and Plutonium

The credit for discharged uranium is calculated as the value of producing uranium of equal enrichment for fabrication, less a penalty for the presence of uranium-236. The poison cost penalty of uranium-236 has been estimated to be $1/gm of uranium-236 in the fuel and the enrichment penalty is discussed in Reference [10].

The Present Worth Technique and Levelized Fuel Cycle Cost Equation

The entire nuclear fuel cycle requires expenditures and receipts over a period of approximately five years. It is important to consider, therefore, the alteration of the

value of money as it is exchanged between the utility, customers, and creditors during the fuel cycle time period. The present value of money is always greater than an identical amount in the future, because the money may earn a rate of return while invested elsewhere during the interim. A common method used to take into account the changing worth of money is the present worth technique, which refers the different expenditures and receipts to one point in time by adjusting them to reflect the potential effective earning power during the interim periods.

The levelized fuel cycle cost is that constant charge during the fuel cycle time period necessary to the consumer to meet all expenses associated with the fuel cycle. It is obtained by requiring that the indebtedness for fuel cycle investments be reduced to zero at the end of the fuel cycle time period. The levelized fuel cycle cost equation generalized for continuous discounting as used in this study is[22]

$$c = \frac{\int_0^n e^{-mr'} \left[\frac{I(m)}{1-\tau} - \left(\frac{\tau}{1-\tau} \right) F_d(m) \right] dm}{\int_0^n e^{-mr'} \cdot Q(m)\, dm} \qquad (6)$$

where

$I(m)$ = investments during the mth month (mills/month-kg)

$F_d(m)$ = depreciation during the mth month (mills/month-kg)

$Q(m)$ = quantity of electricity generated (Kwh(e)/

Kg-month)

τ = income tax rate

r' = effective rate of return

n = length of the fuel cycle (months)

In-core Physics

In conducting an economic study of a reactor fuel cycle it is necessary to calculate the isotopic composition of the reactor fuel discharged and to assure that the reactor is critical. In this study a zero-dimensional modified one group model was used.

It has been noted by Dawson that the Maxwellian flux approximation is inadequate for plutonium-uranium-water lattices [3]. This is because there are large resonances in the low eV region for plutonium 239, 240, and 241. The energy dependent flux depends on the location and width of these resonances, and on the concentration of the individual isotopes of plutonium. Two models which have been used to describe the energy dependent flux are the Wigner-Wilkins model and the Nelkin model. The Nelkin model is a water kernel model and more accurately approximates the scattering properties of water than the Wigner-Wilkins model.

In this study the buildup of isotopes in the reactor were found by obtaining solutions to a set of coupled, differential equations similar to those presented in Reference [2]. The depletion and buildup of various isotopes

were calculated as the summation of the products of the appropriate spectrum averaged cross sections and the thermal flux. Resonance capture in uranium-238 and plutonium-240 was considered by summing the total number of fast neutrons produced in a generation and multiplying by the respective resonance escape probability. In the equations it is assumed that resonance capture in uranium-238 occurs at higher energies than that in plutonium-240. The resonance escape probability for plutonium-240 was calculated from experimental values of the resonance integral for various concentrations of this isotope [3]. This is necessary since self-shielding occurs as plutonium-240 builds up in the reactor. The differential equations used for the concentration of the isotopes in this study are the following:

Uranium-235

$$\frac{dN_{25}}{dt} = -N_{25}\overline{\sigma}_{a_{25}}\phi_T \tag{7}$$

Uranium-236

$$\frac{dN_{26}}{dt} = N_{25}\overline{\sigma}_{\gamma_{25}}\phi_T - N_{26}\overline{\sigma}_{a_{26}}\phi_T \tag{8}$$

Uranium-238

$$\frac{dN_{28}}{dt} = -N_{28}\overline{\sigma}_{a_{28}}\phi_T - \varepsilon P_1(1-p_{28})\{N_{25}\overline{\eta_{25}\sigma}_{a_{25}}\phi_T + N_{49}\overline{\eta_{49}\sigma}_{a_{49}}\phi_T + N_{41}\overline{\eta_{41}\sigma}_{a_{41}}\phi_T\} \tag{9}$$

Plutonium-239

$$\frac{dN_{49}}{dt} = N_{28}\bar{\sigma}_{a_{28}}\phi_T + \varepsilon P_1(1-p_{28})\{N_{25}\overline{\eta_{25}\sigma}_{a_{25}}\phi_T + N_{49}\overline{\eta_{49}\sigma}_{a_{49}}\phi_T + N_{41}\overline{\eta_{41}\sigma}_{a_{41}}\phi_T\} - N_{49}\bar{\sigma}_{a_{49}}\phi_T \quad (10)$$

Plutonium-240

$$\frac{dN_{40}}{dt} = N_{49}\bar{\sigma}_{\gamma_{49}}\phi_T - \varepsilon P_1 p_{28}(1-p_{40})\{N_{25}\overline{\eta_{25}\sigma}_{a_{25}}\phi_T + N_{49}\overline{\eta_{49}\sigma}_{a_{49}}\phi_T + N_{41}\overline{\eta_{41}\sigma}_{a_{41}}\phi_T\} - N_{40}\bar{\sigma}_{a_{40}}\phi_T \quad (11)$$

Plutonium-241

$$\frac{dN_{41}}{dt} = N_{40}\bar{\sigma}_{a_{40}}\phi_T + \varepsilon P_1 p_{28}(1-p_{40})\{N_{25}\overline{\eta_{25}\sigma}_{a_{25}}\phi_T + N_{49}\overline{\eta_{49}\sigma}_{a_{49}}\phi_T + N_{41}\overline{\eta_{41}\sigma}_{a_{41}}\phi_T\} - N_{41}\bar{\sigma}_{a_{41}}\phi_T \quad (12)$$

Plutonium-242

$$\frac{dN_{42}}{dt} = N_{41}\bar{\sigma}_{\gamma_{41}}\phi_T - N_{42}\bar{\sigma}_{a_{42}}\phi_T \quad (13)$$

Fission Product Pairs

$$\frac{dN_{FP}}{dt} = N_{25}\bar{\sigma}_{f_{25}}\phi_T + N_{49}\bar{\sigma}_{f_{49}}\phi_T + N_{41}\bar{\sigma}_{f_{41}}\phi_T \quad (14)$$

where

N_i = atom density of the ith isotope

$\bar{\sigma}_{ai}$ = average microscopic absorption cross section of the ith isotope

$\bar{\sigma}_{\gamma i}$ = average microscopic capture cross section of the ith isotope

$\overline{\sigma}_{fi}$ = average microscopic fission cross section of the ith isotope

$\overline{\eta_i \sigma_{ai}}$ = average product of the microscopic absorption cross section and the number of neutrons produced per neutron absorbed in the fuel

ϕ_T = thermal flux

ε = fast fission factor

P_1 = fast nonleakage probability

p_{28} = resonance escape probability for uranium-238

p_{40} = resonance escape probability for plutonium-240

PROCEDURE

A parametric investigation of the levelized fuel cycle costs was conducted using the algorithm shown in Figure 2 for a model pressurized water reactor whose characteristics were tabulated in Reference [10].

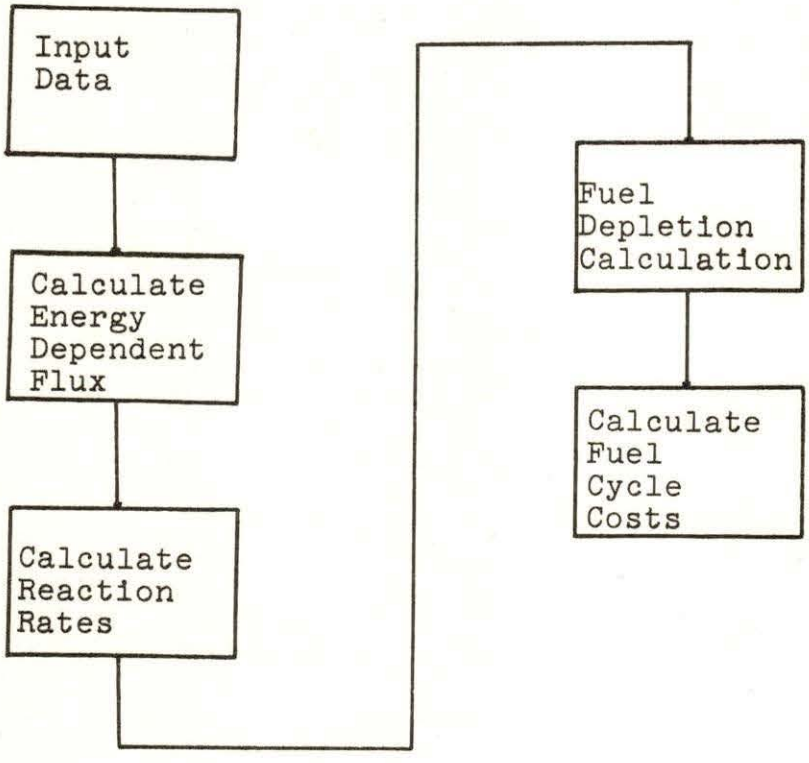

Figure 2. Algorithm of the basic procedure used in calculating the fuel cycle costs

The parameters which were varied in this study were the fabrication penalty for plutonium, the price for fissile plutonium and uranium, the atom percent of uranium-235 in the fuel, the atom percent of plutonium in the fuel, and the

isotopic composition of the plutonium.

In determining the reaction rates, it was necessary to determine the energy dependent flux. In generating a neutron spectrum a graph giving spectra for zero, one, and two percent plutonium in the reactor fuel from the computer code PANTHER [4] was used. The PANTHER code uses a Nelkin kernel to calculate the energy dependent flux and has been correlated with experimental results from the Hanford and Saxton reactors. It is illustrated in Figure 3 that the energy dependent flux approximates a hardened Maxwellian distribution when no plutonium is present in the reactor fuel. A set of ratios, defined as the energy dependent flux when one percent of the reactor fuel is plutonium to the energy dependent flux when no plutonium is present in the reactor fuel, for 100 energy groups was determined from the graph. A second set of ratios, defined as the energy dependent flux when two percent of the reactor fuel is plutonium to the energy dependent flux when no plutonium is present in the reactor fuel, was also determined from the graph for 100 energy groups. Hence to calculate the energy dependent flux, a hardened Maxwellian flux distribution was generated such as would be expected for the model reactor design if there were no plutonium in the reactor fuel, and each element of this flux distribution was multiplied by the appropriate ratio from the first or second set as defined above to give the energy dependent flux when one or two percent of the reactor fuel

was plutonium, respectively. When the concentration was other than one or two percent of the reactor fuel, the ratios were determined by extrapolating or interpolating from the two previously defined sets assuming a linear relationship.

An example of this procedure is shown in Figure 3. If it is desired to know the flux at energy E_i when 1.5 percent

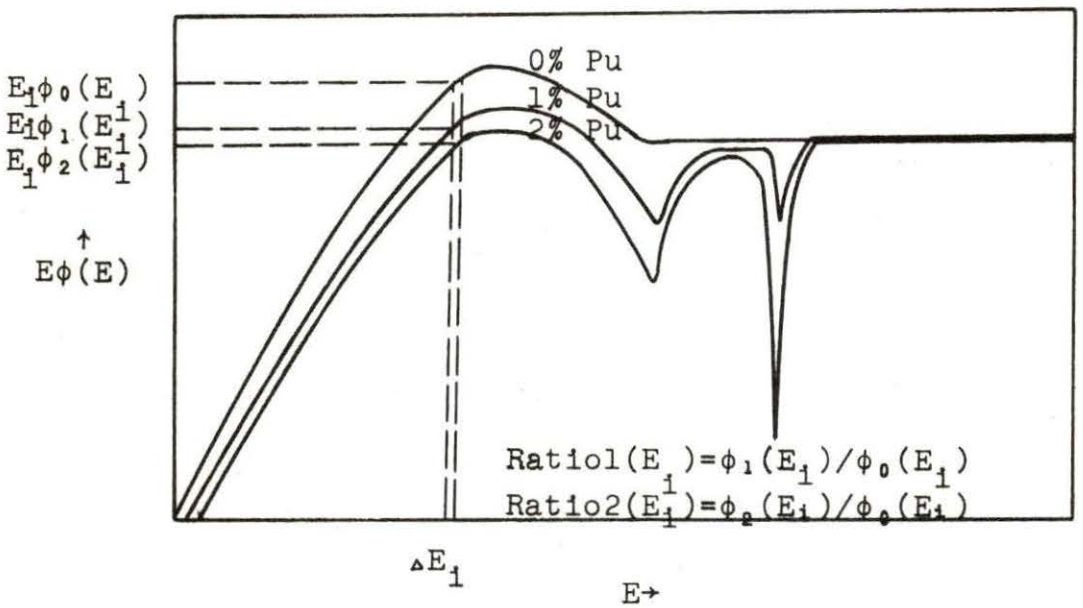

Figure 3. The energy dependent flux for different concentrations of plutonium in the reactor fuel (illustrative example)

of the reactor fuel is plutonium, a hardened Maxwellian flux distribution typical of when no plutonium is present is calculated at E_i, $\phi_0(E_i)$, which in turn is multiplied by an appropriate interpolated ratio:

$$\phi_{1.5}(E_i) = \phi_0(E_i)\left\{ \frac{\phi_1(E_i)}{\phi_0(E_i)} + \left[\frac{\phi_2(E_i)}{\phi_0(E_i)} - \frac{\phi_1(E_i)}{\phi_0(E_i)}\right]\frac{(1.5-1.0)}{(2.0-1.0)} \right\} \quad (15)$$

or

$$\phi_{1.5}(E_i) = \phi_0(E_i) \left\{ \text{Ratio1}(E_i) + \frac{[\text{Ratio2}(E_i) - \text{Ratio1}(E_i)]}{(2.0-1.0)} (1.5-1.0) \right\} \quad (16)$$

In addition to forming a set of ratios to indicate how certain concentrations of plutonium might modify the energy dependent flux, graphs giving spectra for different compositions of plutonium were used to consider the effect of plutonium-240 on the flux [15]. This isotope is important because of its large capture resonance at 1.056 eV. A set of ratios, defined as the energy dependent flux when five percent of the plutonium was plutonium-240 to the energy dependent flux when twenty-three percent of the plutonium was plutonium-240, was determined from the graphs for 100 energy groups. By multiplying each element of the energy dependent flux distribution calculated for a given plutonium concentration, with an assumed plutonium-240 concentration of five percent, by the inverse of the appropriate ratio, the energy dependent flux distribution for the same concentration of plutonium in the reactor fuel, but with a plutonium composition of twenty-three percent plutonium-240, was obtained. The appropriate ratios for plutonium with compositions differing from twenty-three percent plutonium-240 were obtained by noting that each element of the ratio

set should be unity when the plutonium composition is five percent plutonium-240, and extrapolating or interpolating from these two compositions assuming a linear relationship.

Since the plutonium compositions used in defining the first two sets of ratios had seven and seventeen percent plutonium-240, respectively, it was necessary to apply a correction factor to each ratio so that it corresponded to a plutonium composition with five percent plutonium-240. This was done by using the set of ratios discussed in the above paragraph.

Figure 4 illustrates the procedure to correct the flux for the presence of plutonium-240. Again consider the ex-

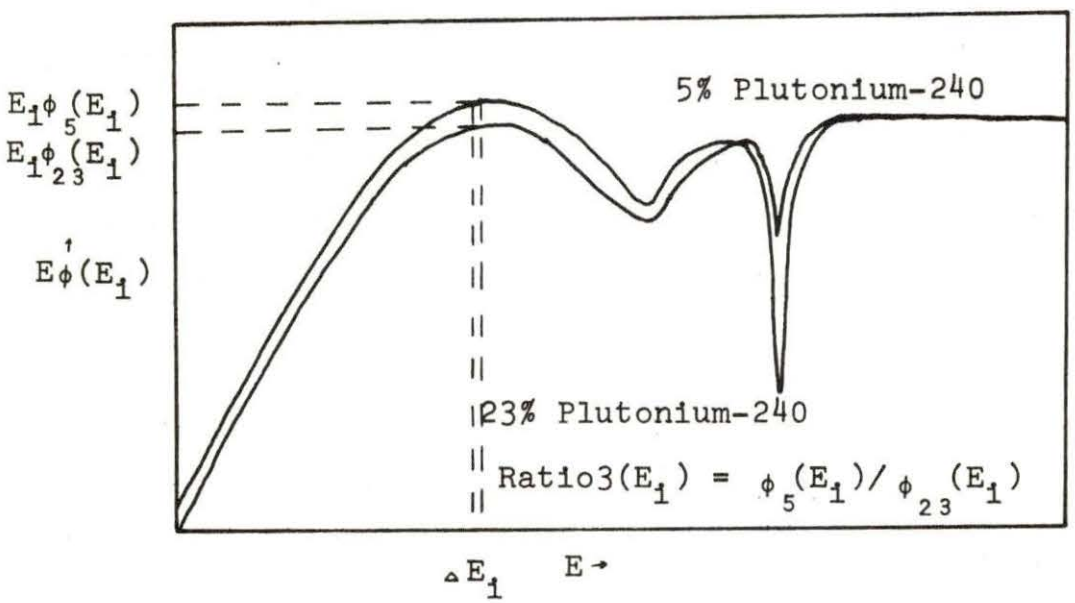

Figure 4. The energy dependent flux for different compositions of plutonium in equal concentrations in the reactor fuel (illustrative example)

ample when the concentration of plutonium in the reactor fuel is 1.5 percent. Assume also that the composition of the plutonium includes 15.0 percent plutonium-240. But $\phi_{1.5}(E_i)$ was calculated assuming that the composition of the plutonium included 5.0 percent plutonium-240. The flux corresponding to a plutonium composition of 15.0 percent plutonium-240 is found by multiplying $\phi_{1.5}(E_i)$ by an appropriate interpolated ratio:

$$\phi_{Pu}(E_i) = \phi_{1.5}(E_i) \left\{ \frac{\phi_5(E_i)}{\phi_5(E_i)} + \frac{\left[\frac{\phi_5(E_i)}{\phi_{23}(E_i)} - \frac{\phi_5(E_i)}{\phi_5(E_i)}\right]}{(23.0-5.0)} (15.0-5.0) \right\}^{-1} \tag{17}$$

or

$$\phi_{Pu}(E_i) = \phi_{1.5}(E_i) \left\{ 1 + \frac{(\text{Ratio3}(E_i) - 1)}{(23.0-5.0)} (15.0-5.0) \right\}^{-1} \tag{18}$$

Once the energy dependent flux was obtained, the reaction rates needed for equations (7) to (14) were computed by numerical integration using Simpson's rule. In this study the thermal energy group consisted of the energy range 0 to 0.5 eV. It was concluded by calculations using 10, 25, 50, and 100 energy groups of equal widths in the thermal region that 100 groups were necessary to accurately consider the depression of the flux in the region where plutonium-239 and -241 have low energy absorption resonances. Cross section data came from References [6] and [8], and doppler

broadening effects were considered by the use of the expressions given in Lamarsh [11] and Beckurts and Wirtz [1]. The neutron temperature was determined by the use of a graph in Lamarsh [11] giving the neutron temperature as a function of the moderator temperature and the amount of absorption per hydrogen atom. Expressions and data presented in the text by Glasstone and Sesonske [5] were used to calculate the resonance escape probability for uranium-238, the fast fission factor, and the fast nonleakage probability.

To determine the change in reaction rates with time in the reactor, the following expression for constant generation of power was used:

$$\int_{E_T} \varepsilon\gamma(E,t)\Sigma_f(E,t)\phi(E,t)dE = \int_{E_T} \varepsilon\gamma(E,0)\Sigma_f(E,0)\phi(E,0)dE \quad (19)$$

where

$\gamma(E,t)$ = recoverable energy

$\Sigma_f(E,t)$ = microscopic fission cross section

$\phi(E,t)$ = energy-time dependent flux

E_T = thermal energy group

ε = fast fission factor

Since the energy dependent flux was determined as:

$$\phi(E,t) = \phi_0(t)f(E)$$

where f(E) represents the ratios used to modify the hardened Maxwellian and ϕ_0 is the 2200 m/s flux, one can write:

$$\phi(E,t) = \phi_0(0)\frac{\phi_0(t)}{\phi_0(0)}f(E) = \phi(E,0)\Omega \quad (20)$$

where

$$\Omega = \phi_0(t)/\phi_0(0)$$

Then

$$\Omega = \frac{\varepsilon \int_{E_T} \gamma(E,0)\Sigma_f(E,0)\phi(E,0)dE}{\varepsilon \int_{E_T} \gamma(E,t)\Sigma_f(E,t)\phi(E,0)dE} \tag{21}$$

Further, the recoverable energy is not strongly energy or time dependent and,

$$\Sigma_f(E,t) = \sum_i N_i(t)\sigma_{fi}(E)$$

Hence,

$$\Omega \simeq \frac{\sum_i N_i(0)}{\sum_i N_i(t)} \tag{22}$$

where $N_i(t)$ is the atom concentration of the ith fissile isotope present in the fuel at time t.

The depletion calculations were performed by solving equations (7) through (14) on the analog computer because of its speed and economy in solving a problem consisting of coupled differential equations. The factor Ω was computed using a division circuit and multiplied into the output of those potentiometers representing reaction rates. The resonance escape probability for plutonium-240 was calculated

from experimentally determined effective resonance integrals given by Dawson [3] for varying concentrations of this isotope, and programmed on the analog computer by using a variable diode function generator. The flux was normalized by properly choosing the ratio of the computer time to real problem time used in scaling the equations. A neutron balance as given by a one group approximation was also programmed into the circuit to aid in interpreting the relative reactivities of the fuel combinations used. The depletion calculations were recorded for several combinations of initial fuel loadings and burnups of 20,000, 30,000, and 40,000 Mwd/MTU.

The fuel cycle costs were then calculated using the levelized fuel cycle cost equation with straight line depreciation. The fabrication penalty was taken as 30, 60, and 90 percent for recycling plutonium. The prices of plutonium ranged from \$3 to 18/gram-fissile, while that of uranium was chosen as \$8 to 12/pound of U_3O_8. Other input parameters were chosen as shown in Table 1.

Table 1. Fuel cycle cost input parameters

Item	Rate	Begin* Payment (Months)	End Payment (Months)
Purchase of Fuels	$8-12/lb U_3O_8 $3-18/gm Pu fissile	0	0
Conversion†	$1.25/lb U_3O_8	0	3
Enrichment	$32/swu	6	6
Fabrication†	$70/KGU	3	13
Credit for Excess		13	13
Depreciation of Fuel		14	51.5
Cooling of Fuel		51.5	57.5
Transport of Fuel	$ 5/KGH	57.5	57.5
Reprocessing†	$35/KGH	57.5	59.5
Credit for Excess		59.5	59.5
Income Tax Rate	.5		
Effective Rate of Return	.07		

* Burnup = 30,000 Mwd/MTU

† Assumed linear payments

RESULTS

The results of this study are presented as a series of graphs of the levelized fuel cycle cost as a function of the market price of plutonium. From these graphs it is easy to determine the price of plutonium at which it would be economical to recycle the given plutonium-uranium fuel combination.

The notation used on the graphs for the various isotopic combinations of plutonium is given in Table 2. The broken line represents the levelized fuel cycle costs for uranium as the fuel with an enrichment of 3.3 percent uranium-235. Other important information is given on the graphs.

To properly draw conclusions from the graphs concerning the value of a given recycle fuel mixture, it is necessary to know the relative reactivities at a given burnup of fuel. The results of the one-group neutron balance calculations programmed on the analog are therefore presented in Table 3 for burnups of 30,000 Mwd/MTU. It should be mentioned that to prevent saturation of amplifiers on the analog, it was necessary to scale down the voltages representing the concentrations of various isotopes in the reactor. Hence the precision of this calculation may be less than that of the depletion calculations.

Figures 5 through 10 illustrate the levelized fuel cycle costs for fuel mixtures containing different concentrations of plutonium replacing the uranium-235. Except for small

Table 2. Isotopic compositions of the plutonium selected for this study

Designation	Atom Percent Pu-239 in the Pu	Atom Percent Pu-240 in the Pu	Atom Percent Pu-241 in the Pu	Atom Percent Pu-242 in the Pu	Approximate fuel burnup to produce this Pu combination (Mwd/MTU)
A	80.0	15.0	5.0	0.0	9000
B	65.0	23.0	10.5	1.5	23000
C	55.0	27.0	13.0	5.0	32500
D	40.0	34.0	17.5	8.5	>32500
E	20.0	30.0	35.0	15.0	>32500

Table 3. Neutron balance of the various fuel mixtures studied at 30,000 Mwd/MTU

Atom Percent Plutonium in the Fuel	Atom Percent Uranium-235 in the Fuel	Isotopic Composition of Plutonium	Neutron Balance (x constant)
0.0	3.3	(Reference)	31
0.5	2.8	A	76
		B	65
		C	62
		D	50
		E	33
1.0	2.3	A	87
		B	71
		C	65
		D	46
		E	25
1.0	2.6	A	101
		B	85
		C	79
		D	61
		E	39
1.5	1.8	A	99
		B	80
		C	66
		D	46
		E	21
2.0	1.3	A	97
		B	77
		C	62
		D	15
		E	40
2.0	1.6	A	106
		B	86
		C	71
		D	49
		E	24

Table 3. (Continued)

Atom Percent Plutonium in the Fuel	Atom Percent Uranium-235 in the Fuel	Isotopic Composition of Plutonium	Neutron Balance (x constant)
2.5	0.8	A	83
		B	65
		C	52
		D	34
		E	11
3.0	0.3	A	61
		B	48
		C	39
		D	24
		E	7
3.0	0.6	A	66
		B	52
		C	42
		D	28
		E	11

concentrations of plutonium, recycle fuels from discharged fuel that had a long burnup appear more economical for a constant concentration of uranium. The major contribution to reduced fuel cycle costs for these plutonium mixtures was the reduction in the purchase investment, since the plutonium price was determined by the fissile content. However, from Table 3 it should be noted that the reactivity at discharge was also less for these fuels. Hence the total energy available from these fuels is sometimes less than the reference uranium fuel cycle considered.

The graphs also show that the slopes of the cost curves

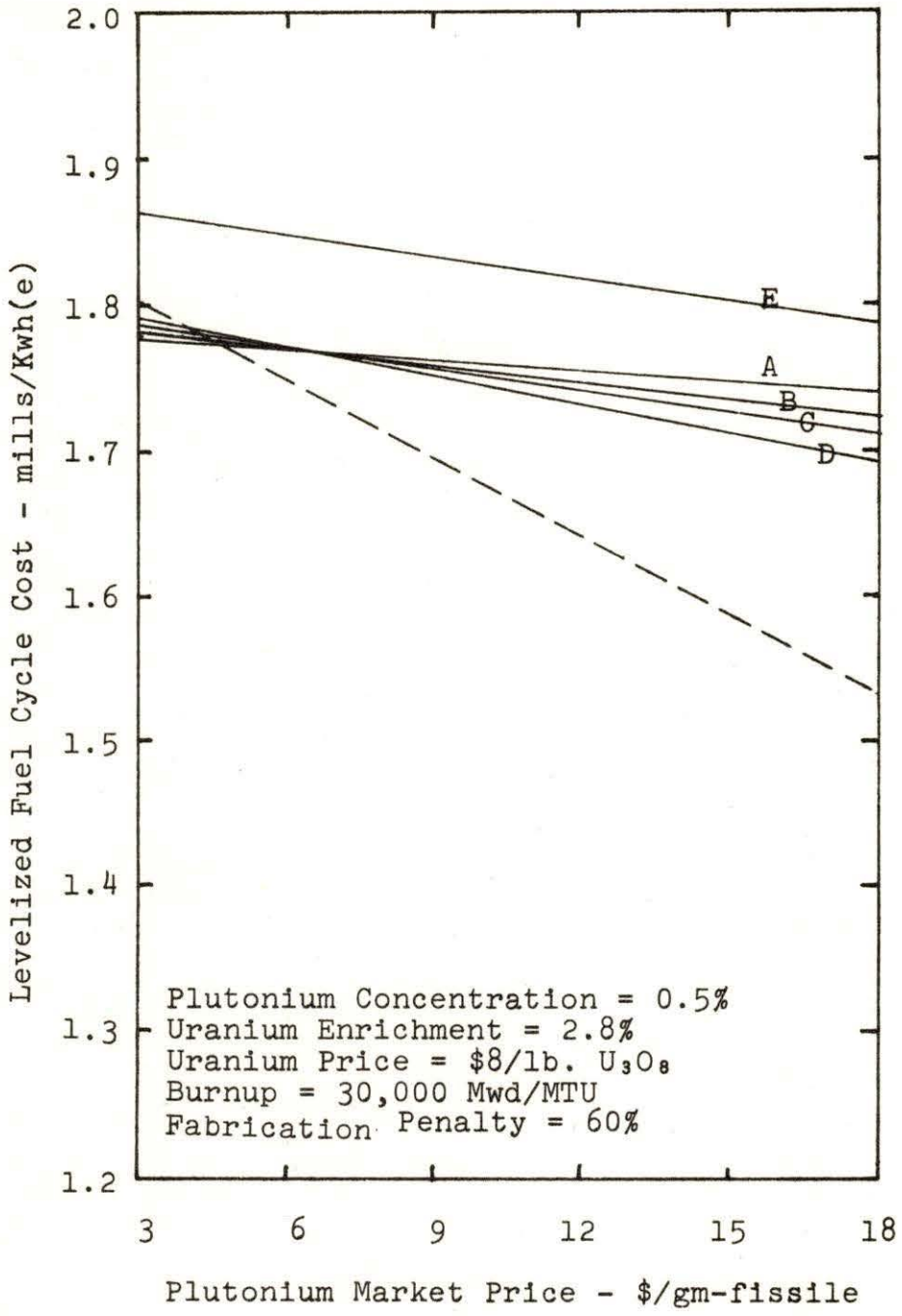

Figure 5. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

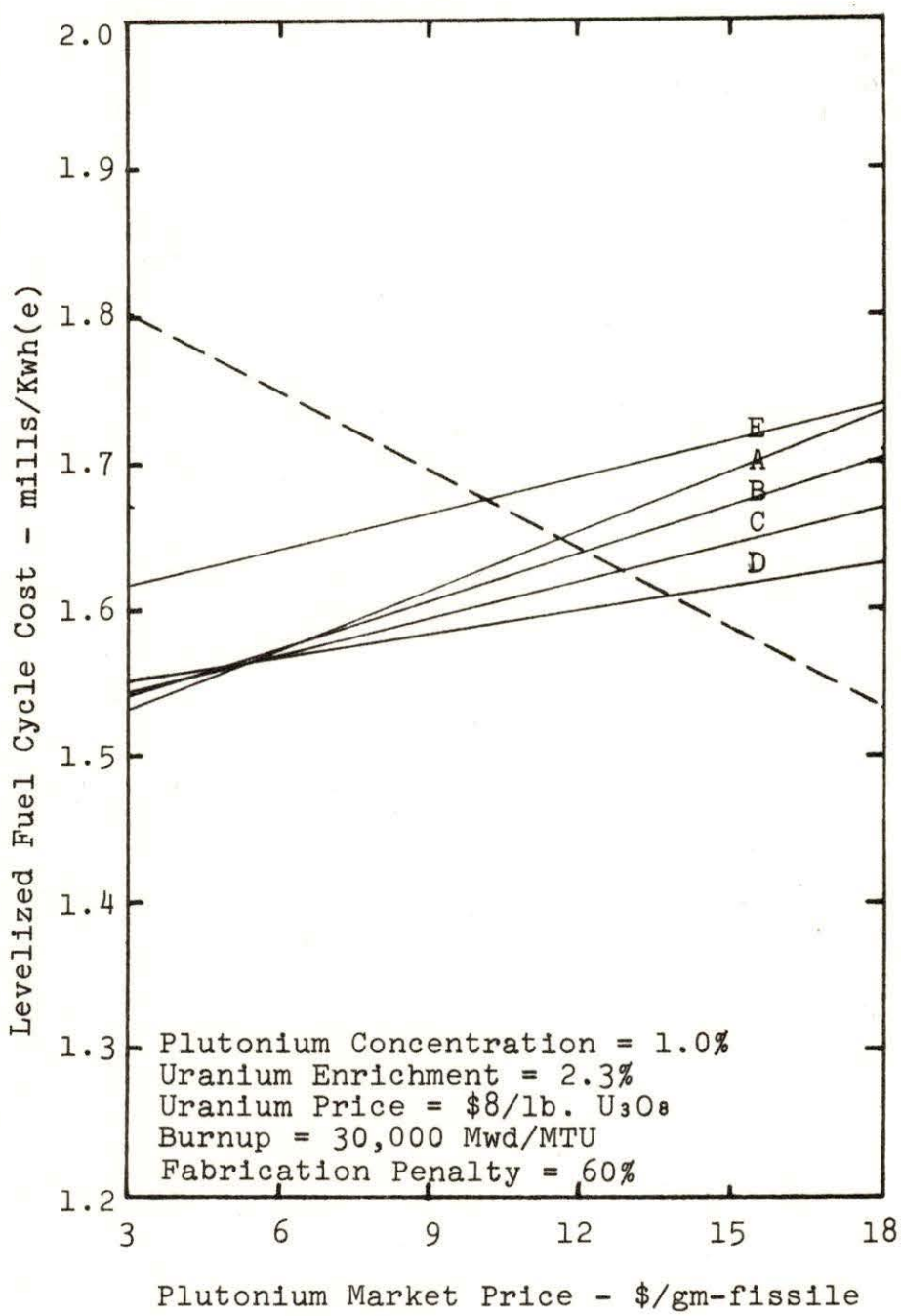

Figure 6. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

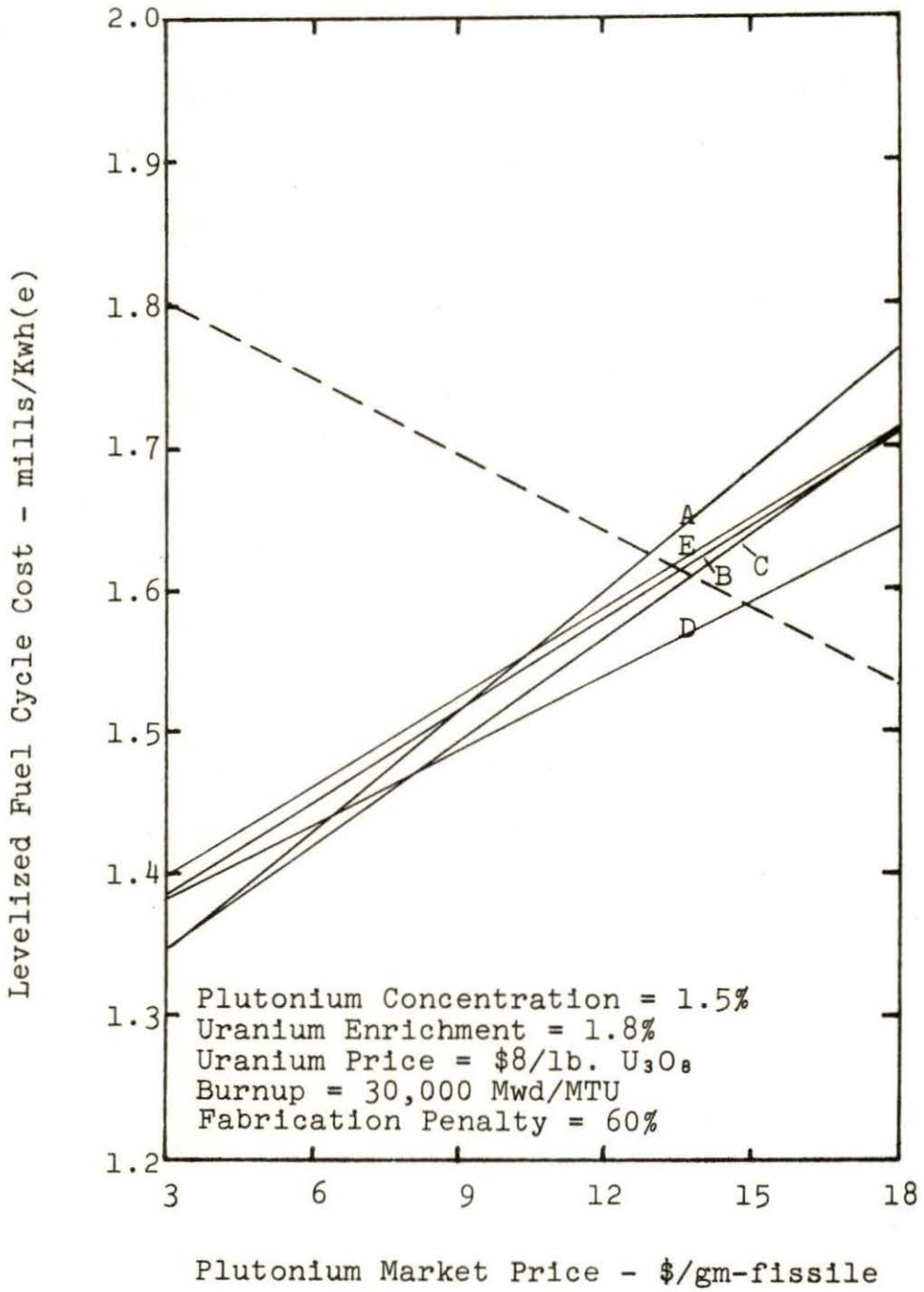

Figure 7. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

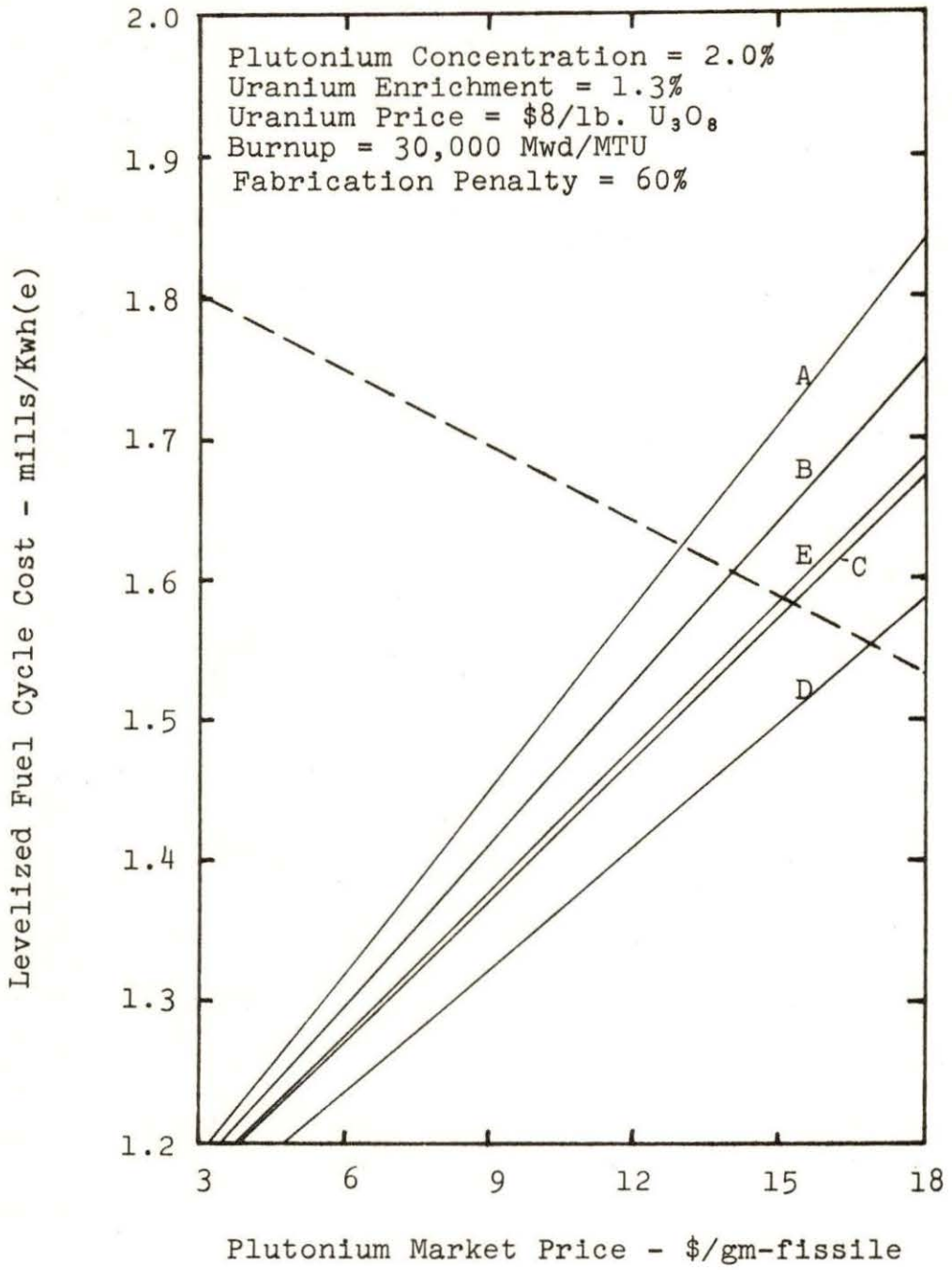

Figure 8. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

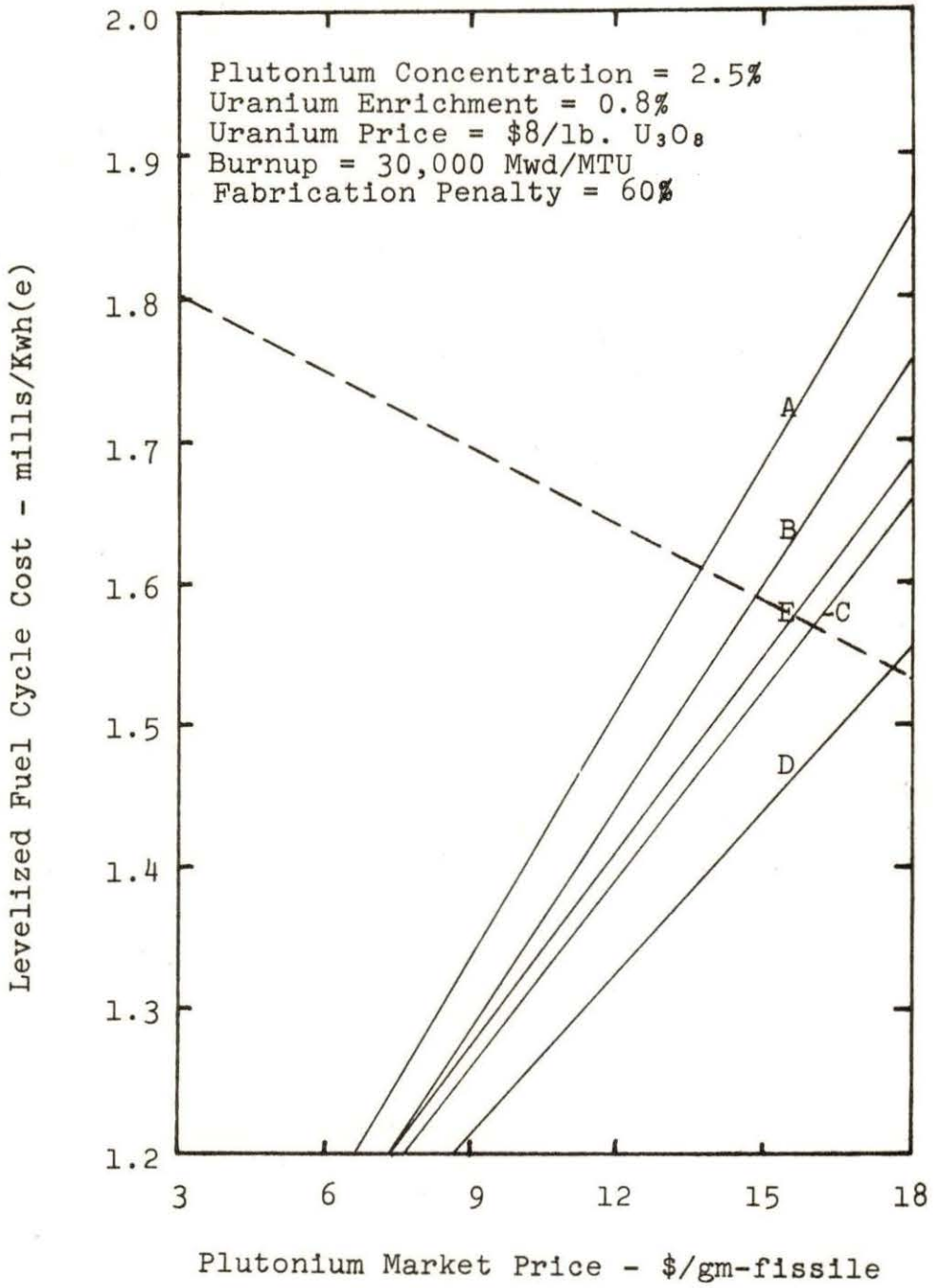

Figure 9. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

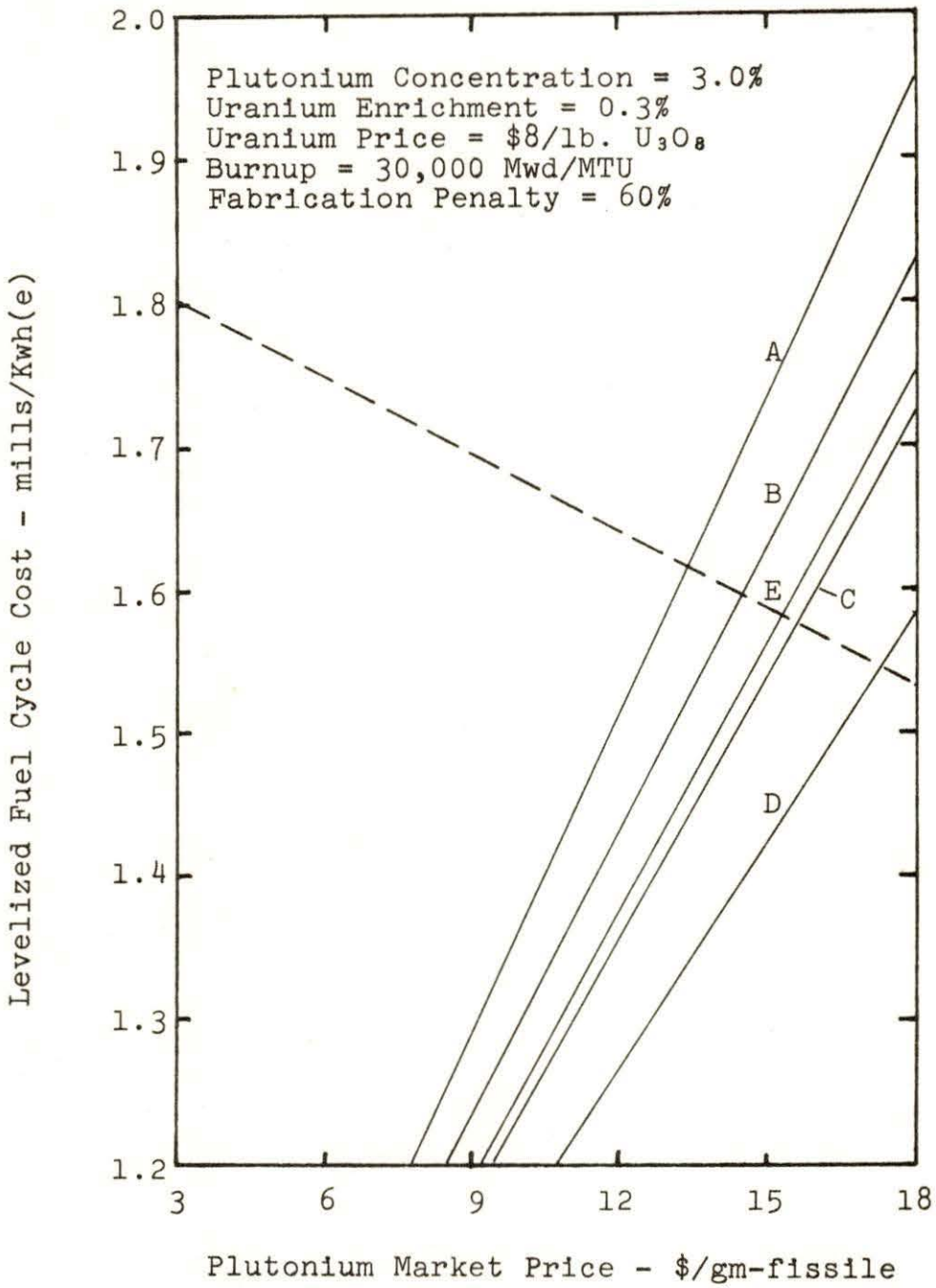

Figure 10. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

change with plutonium concentration. This is better illustrated in Figures 11 and 12 where the plutonium composition is constant, while the concentration is varied. As the concentration of plutonium increases, the reactivity at discharge as given in Table 3 decreases. Hence these figures indicate a mixture of 0.8 percent uranium-235 and 2.5 percent plutonium in the fuel will be the most economical as the price of plutonium falls to a value permitting recycle. A further decrease in the price, however, may result in a mixture of plutonium and depleted uranium as the most economical.

Figures 13 through 18 give the levelized fuel cycle costs for fabrication penalties of 30 and 90 percent. It may be concluded from these figures that fuel costs with small amounts of plutonium are more greatly affected by the fabrication penalty and may be entirely uneconomical.

Figures 19 through 21 illustrate the fuel cycle costs for a greater replacement of plutonium per uranium atom in the fuel, a possibility for actual recycle. The fuel cycle costs are greater, but the reactivity at discharge is also greater.

The results for a price of uranium of \$12/lb. U_3O_8 are illustrated in Figures 22 through 24. Figures 25 through 27 illustrate how the fuel cycle costs depend on burnup.

To determine the value of a given plutonium composition, it is necessary to set the uranium enrichment in the fuel

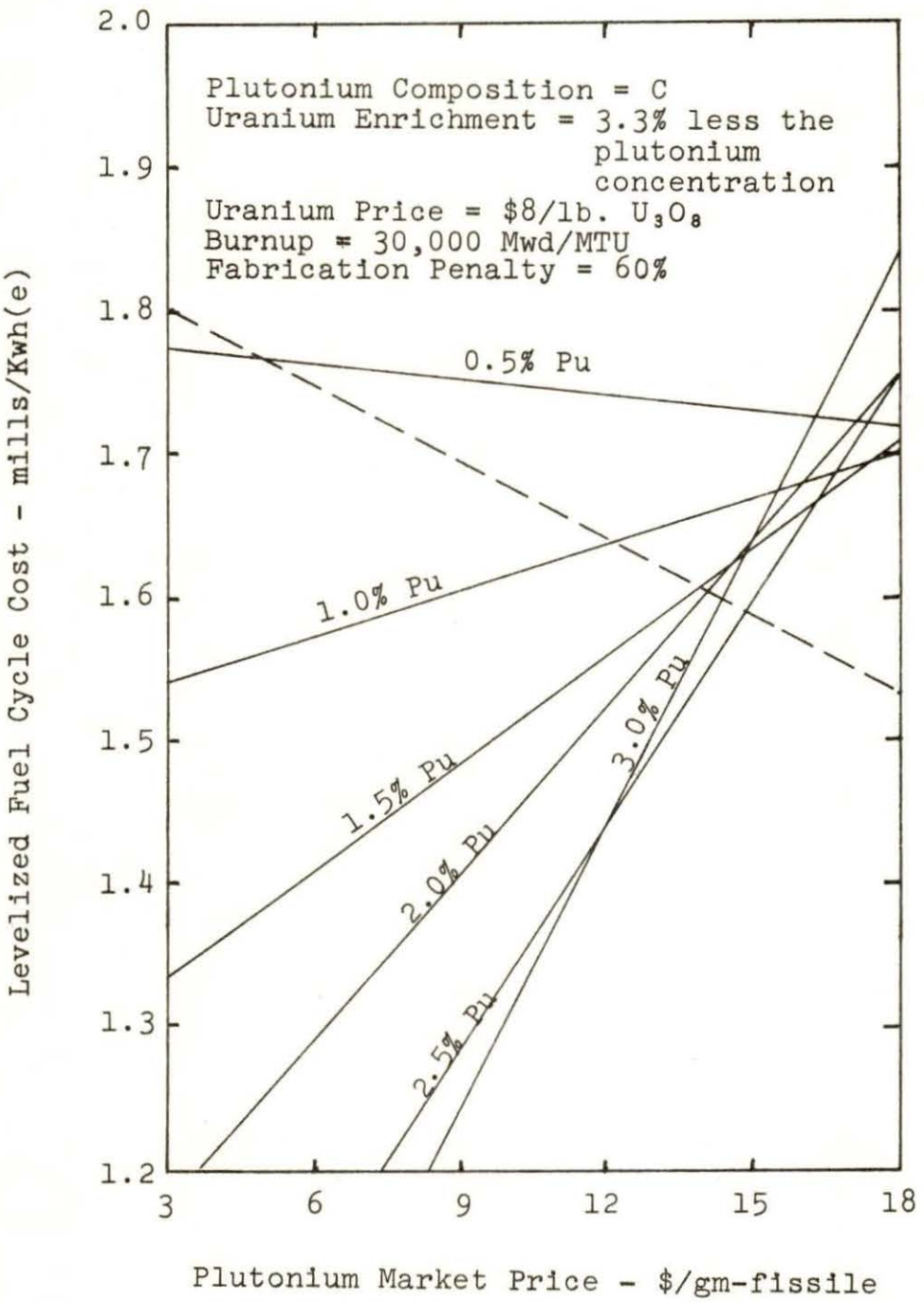

Figure 11. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium concentration

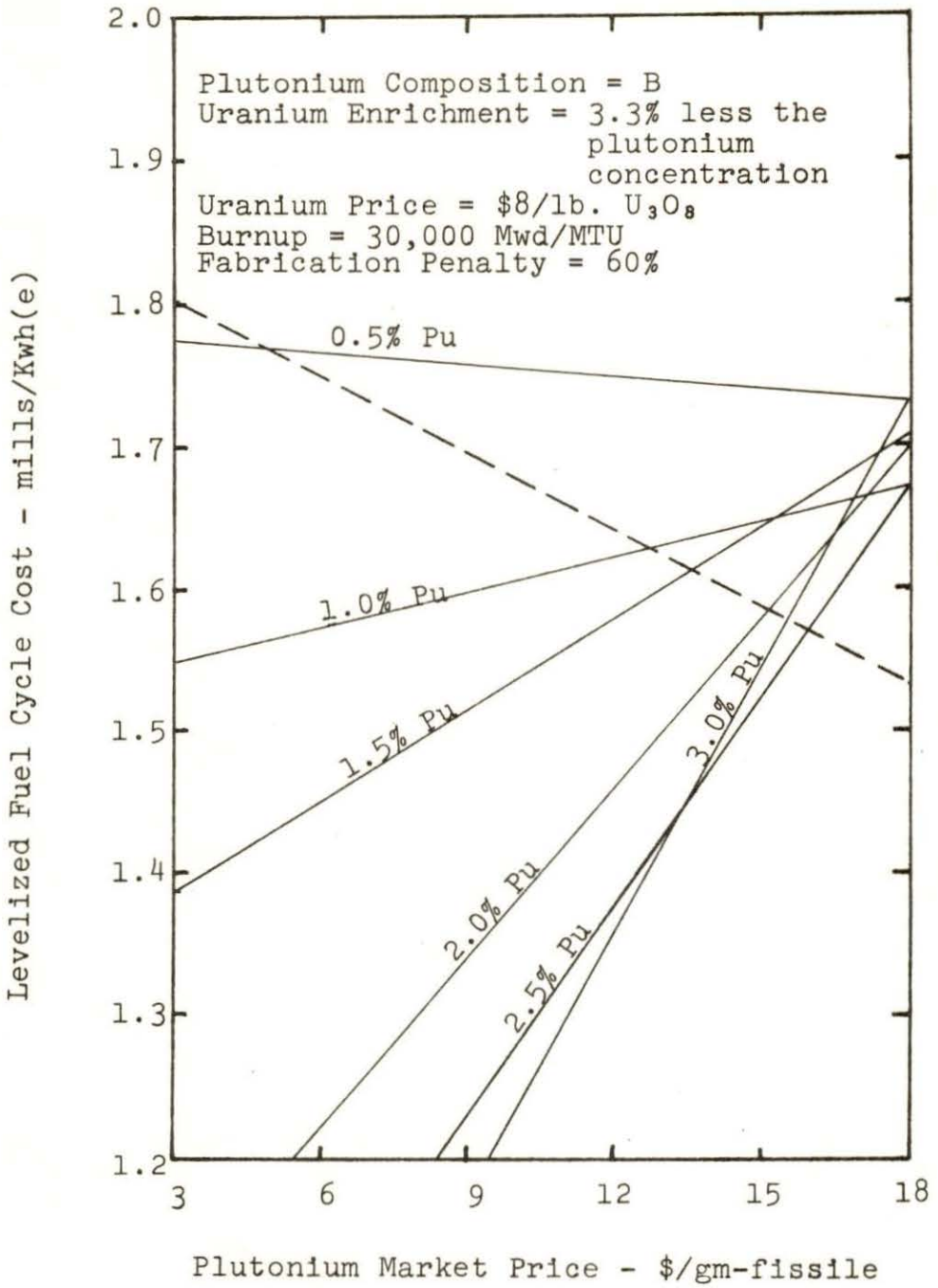

Figure 12. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium concentration

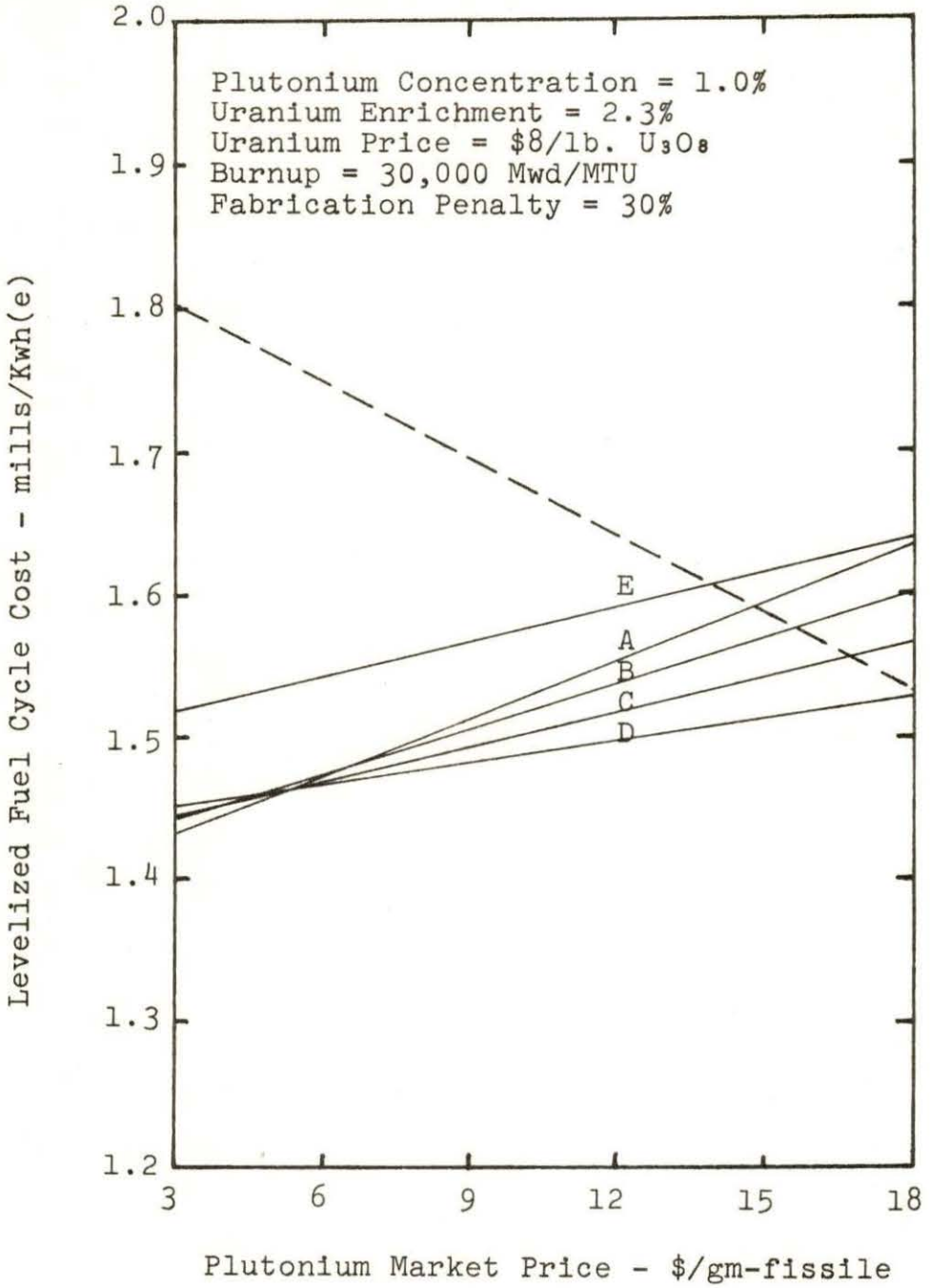

Figure 13. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

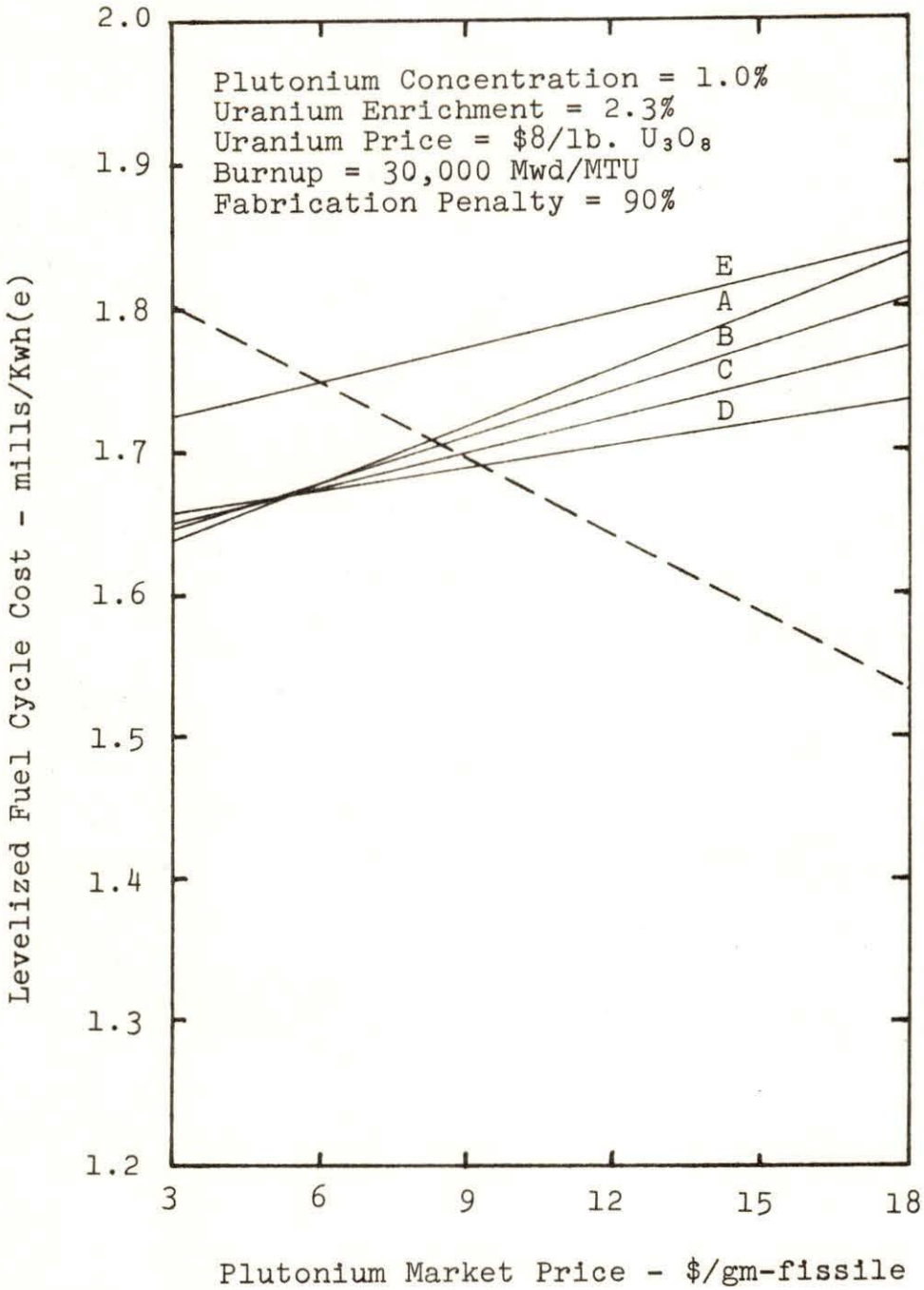

Figure 14. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

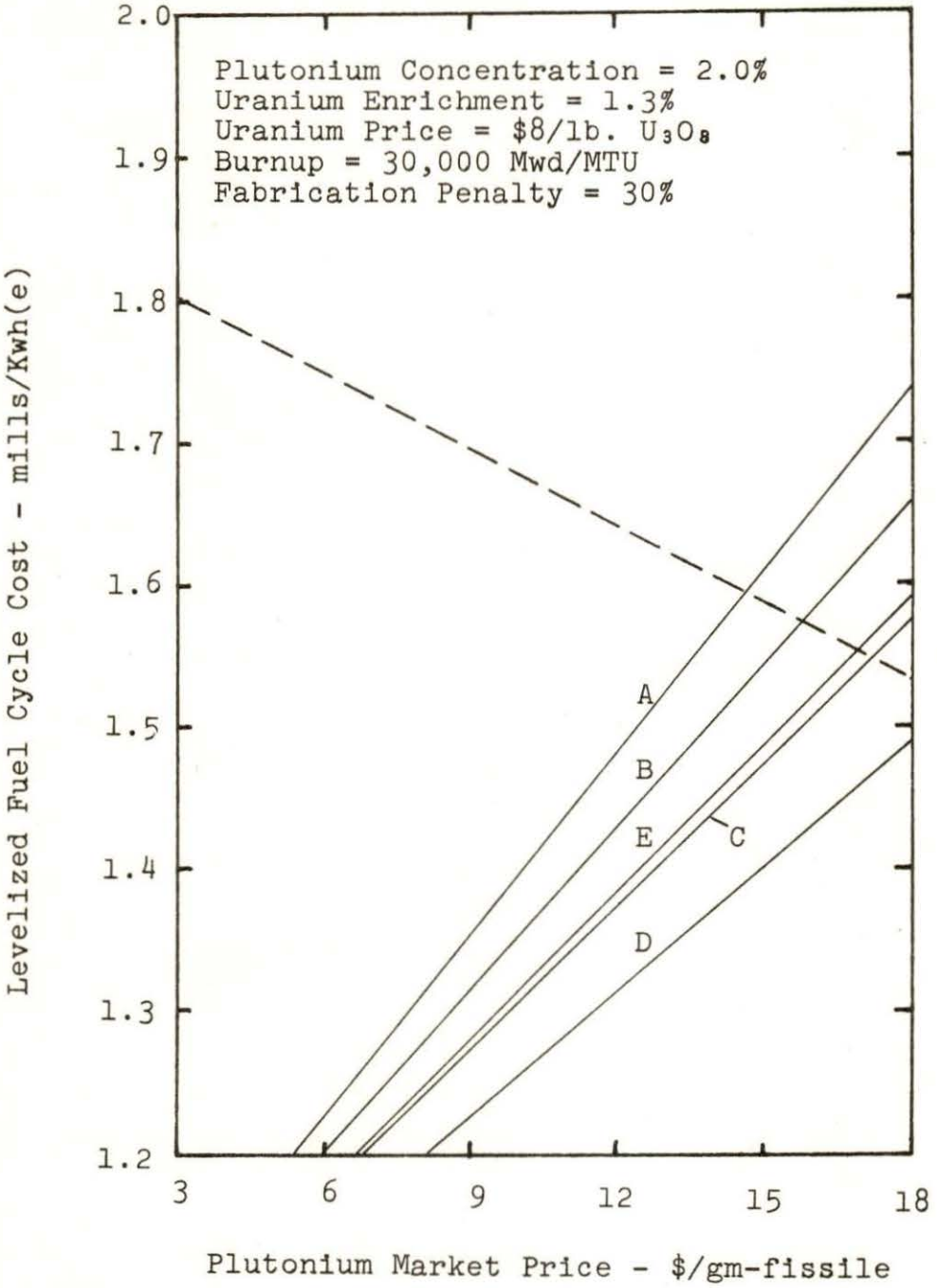

Figure 15. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

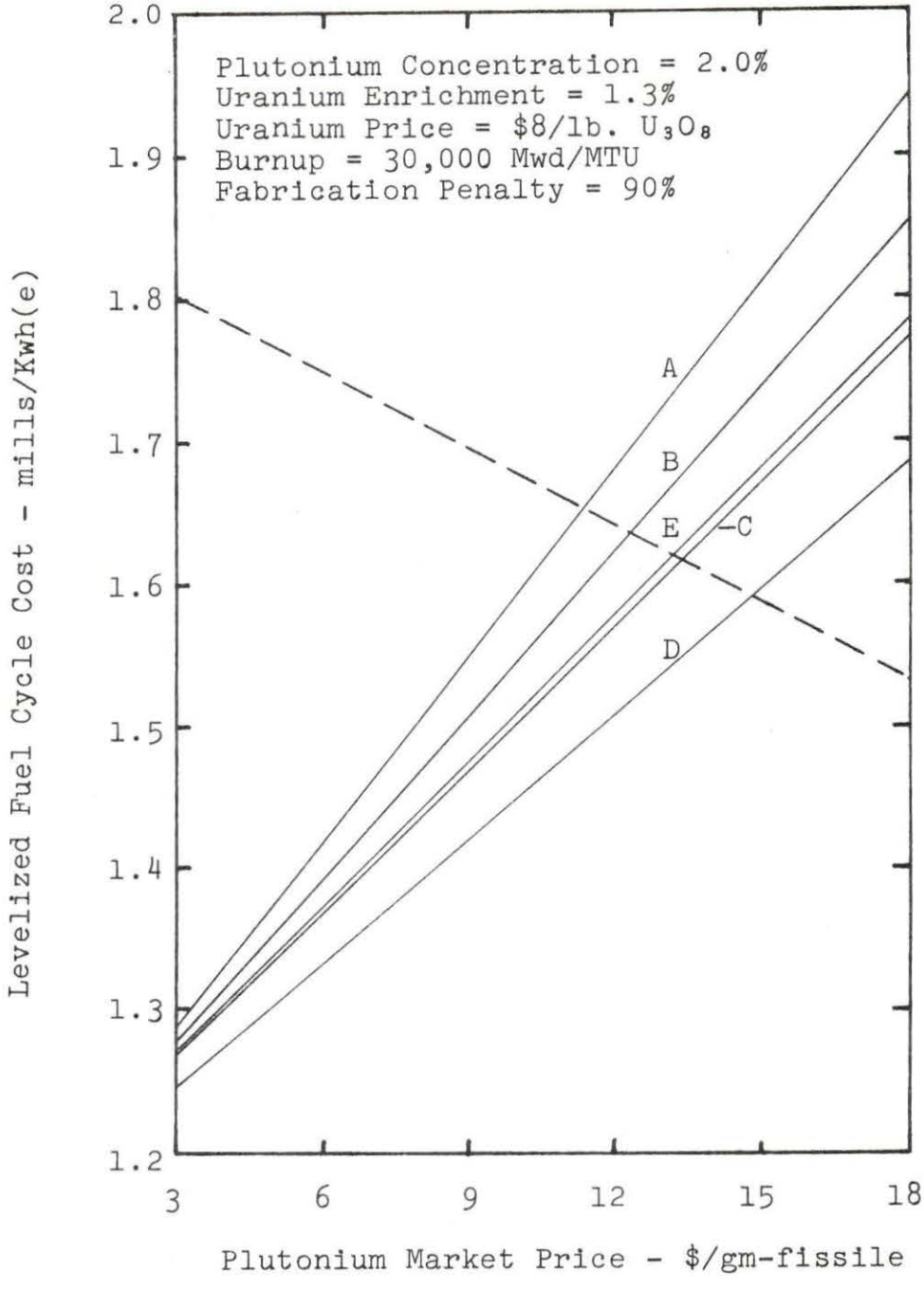

Figure 16. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

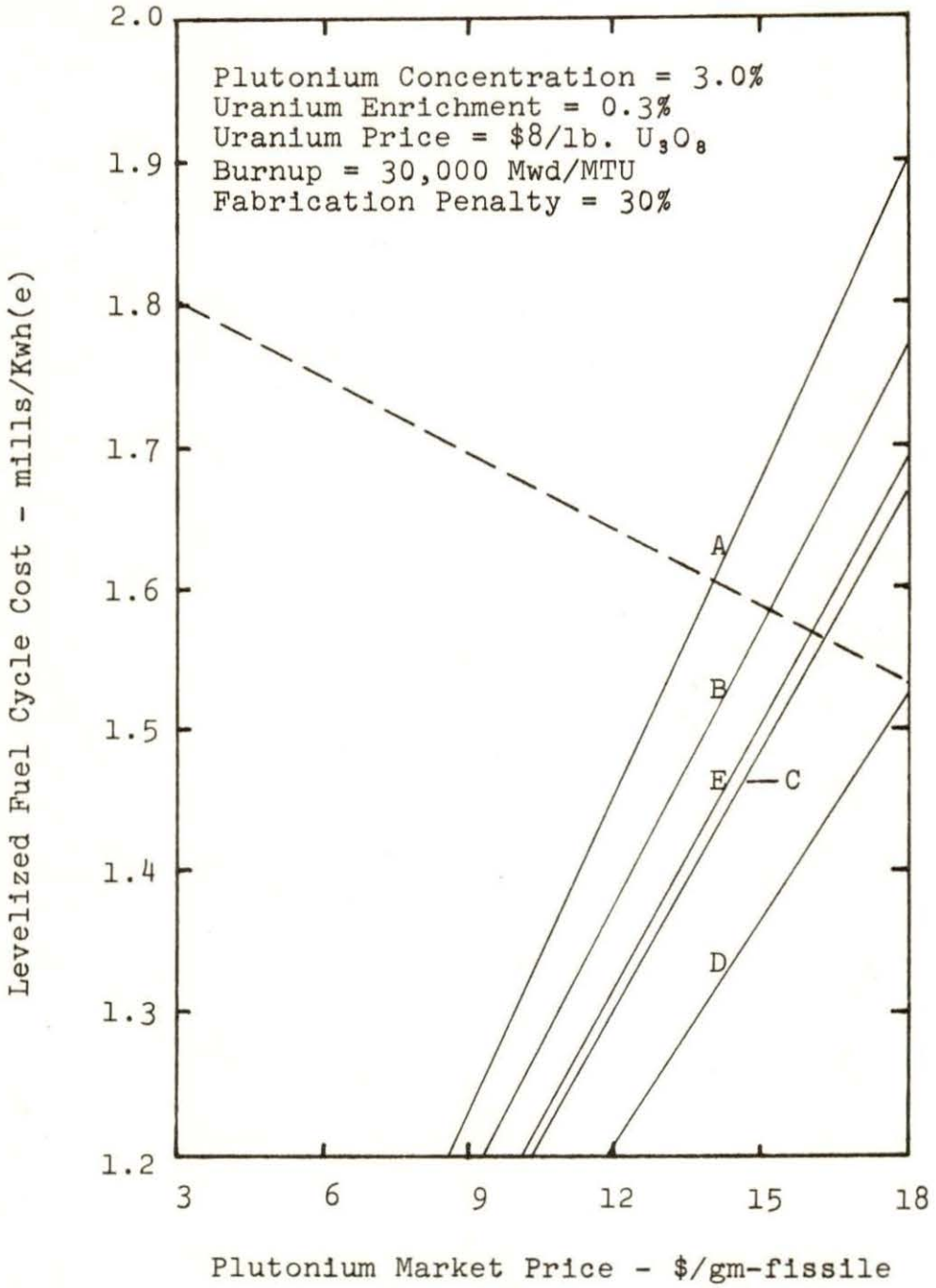

Figure 17. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

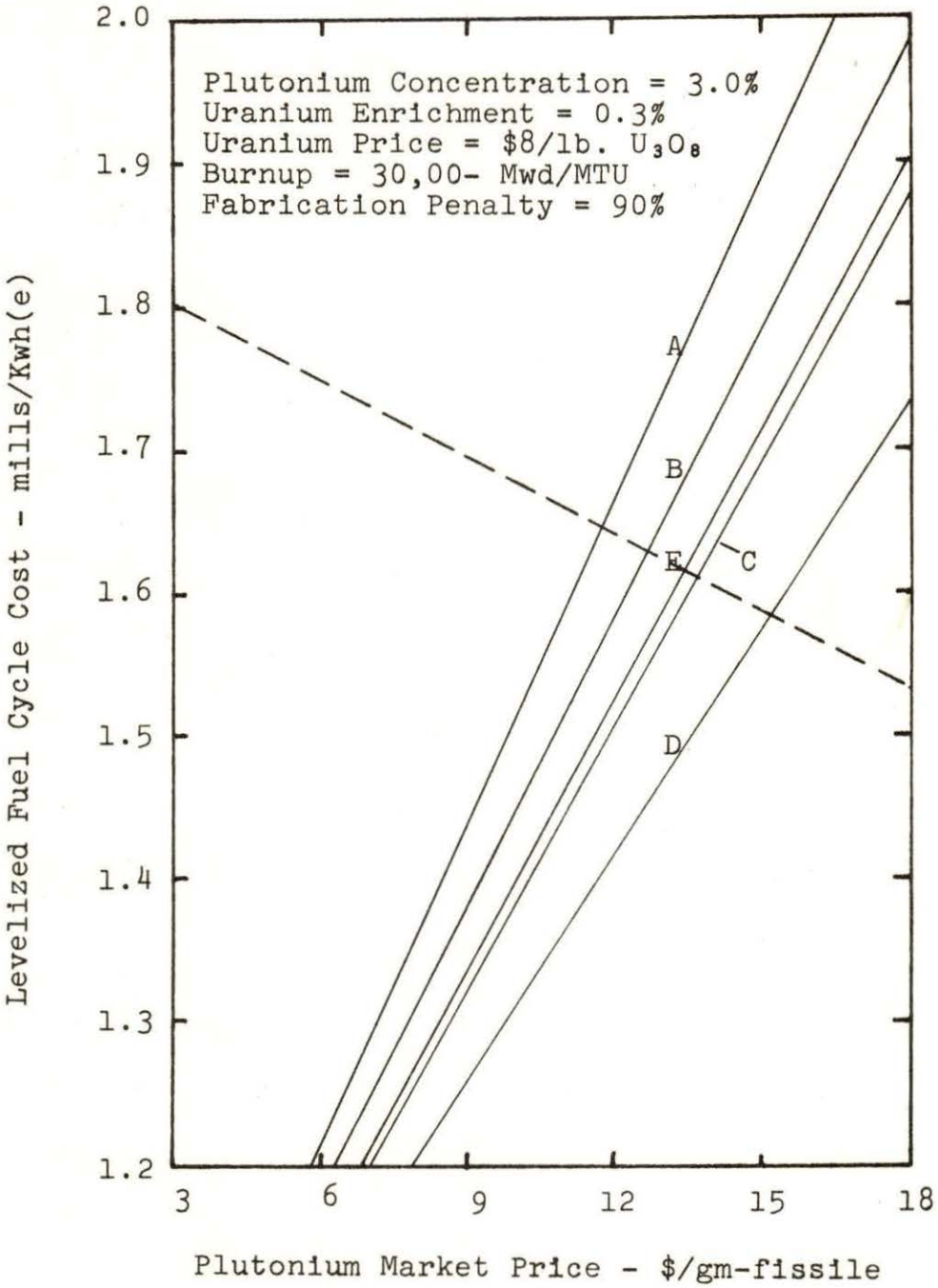

Figure 18. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

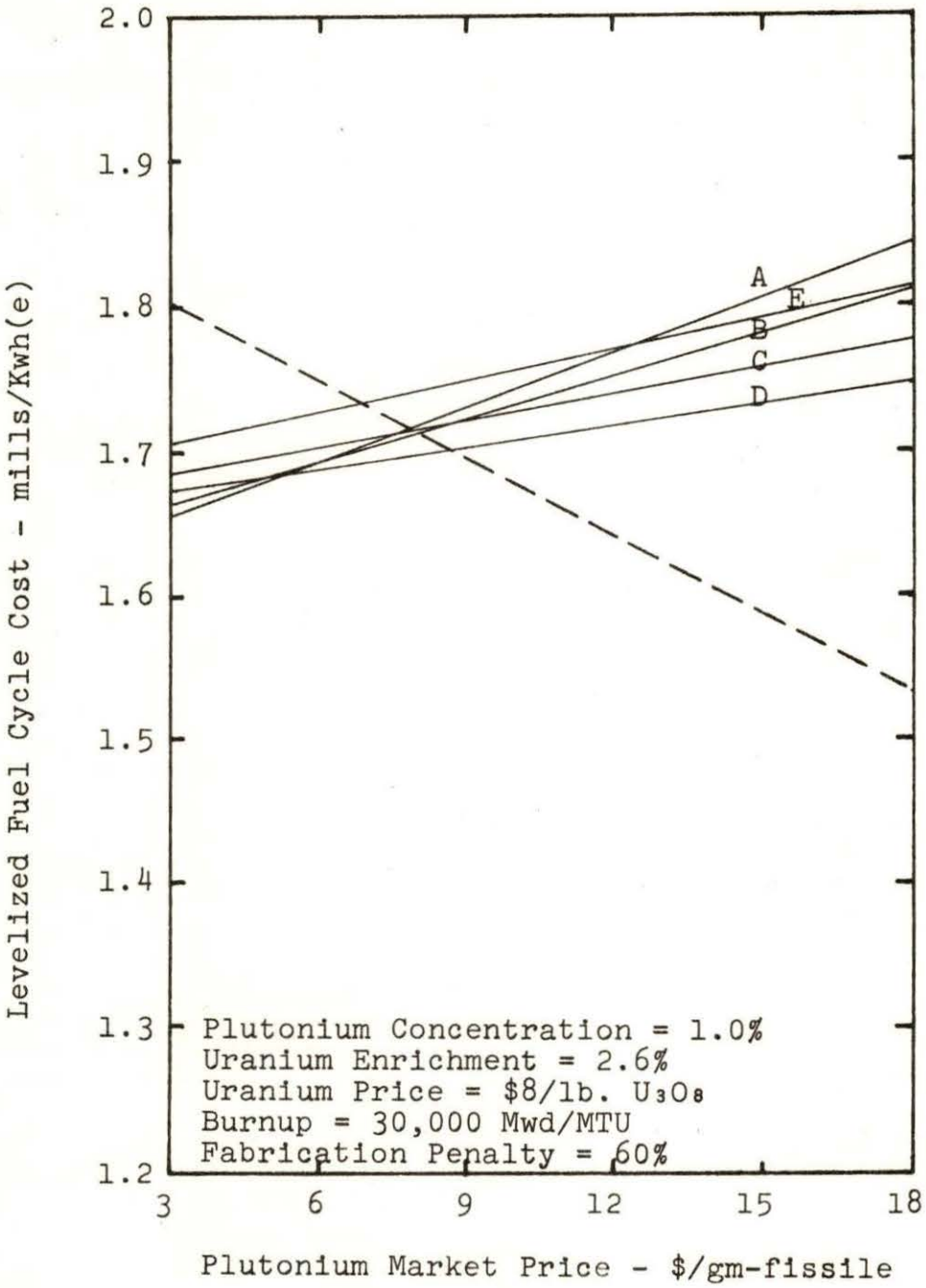

Figure 19. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

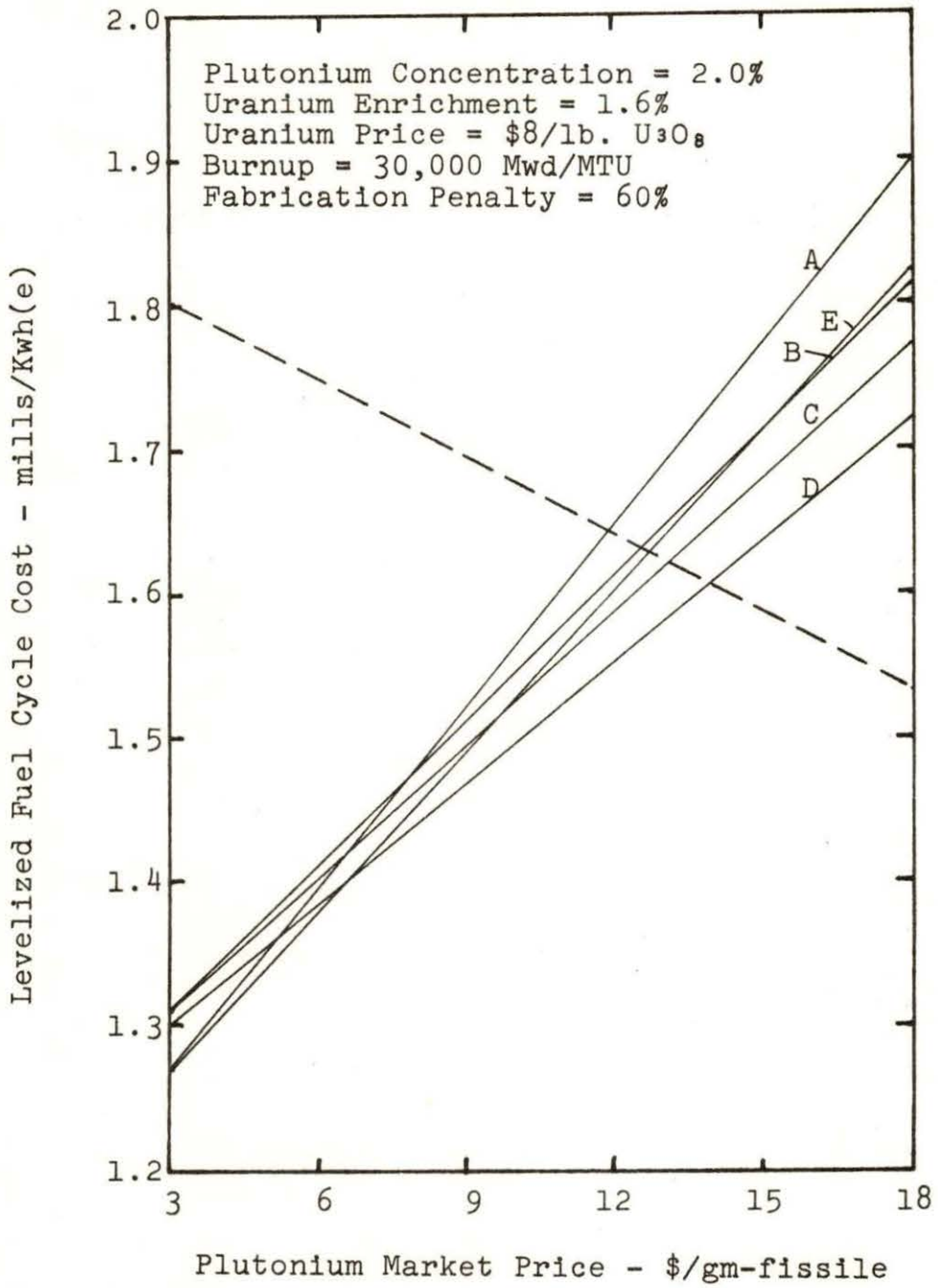

Figure 20. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

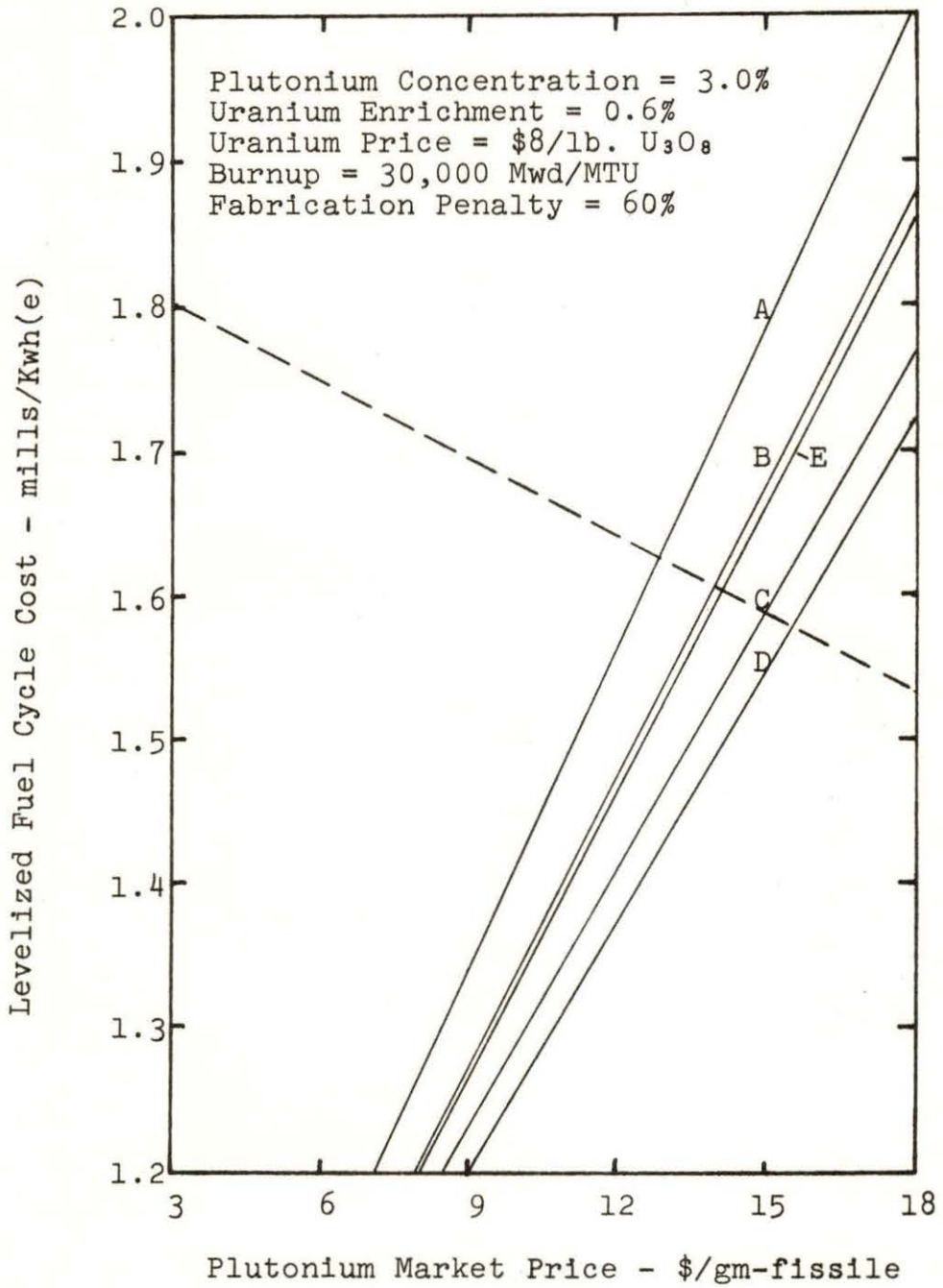

Figure 21. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

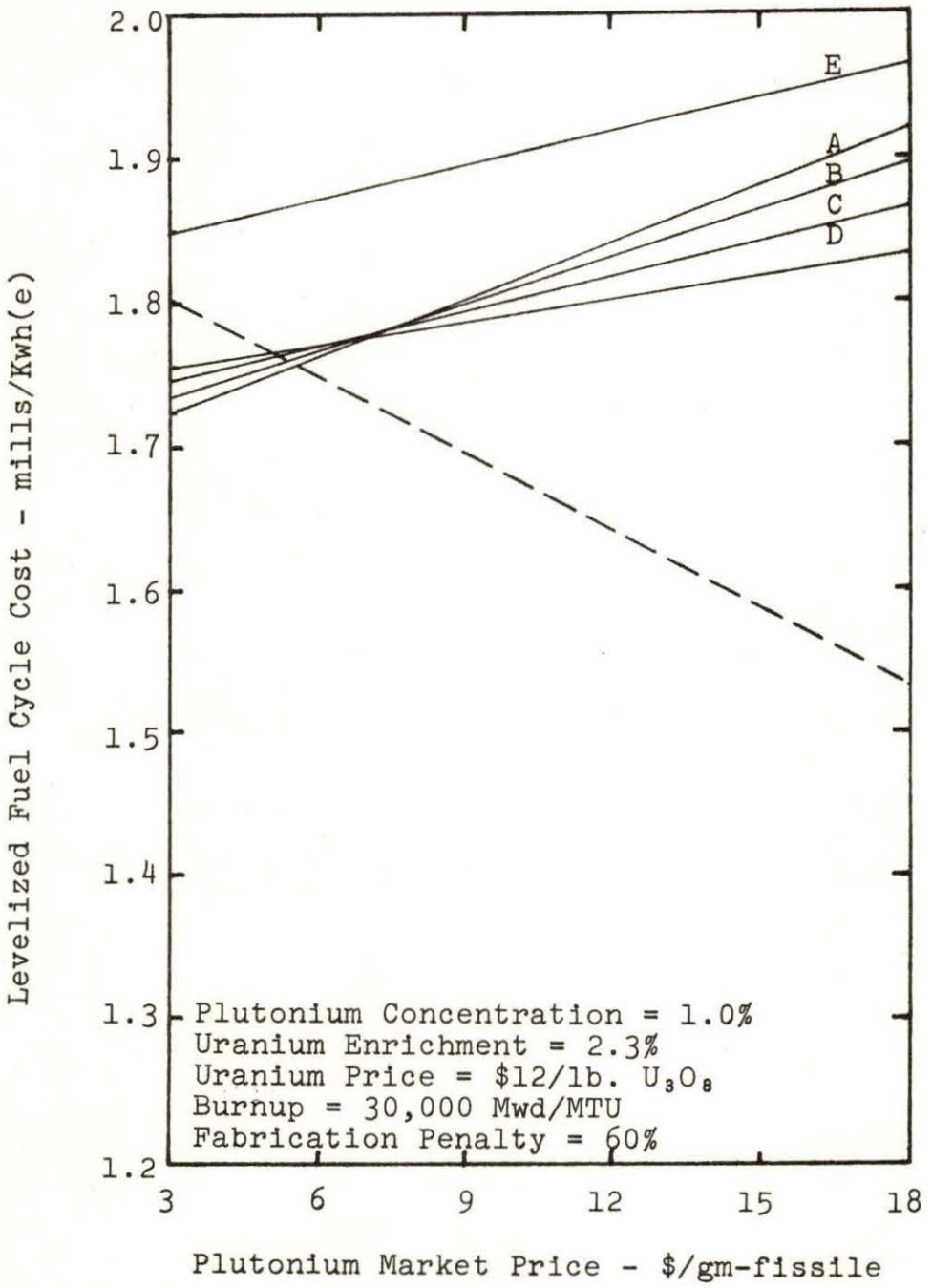

Figure 22. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

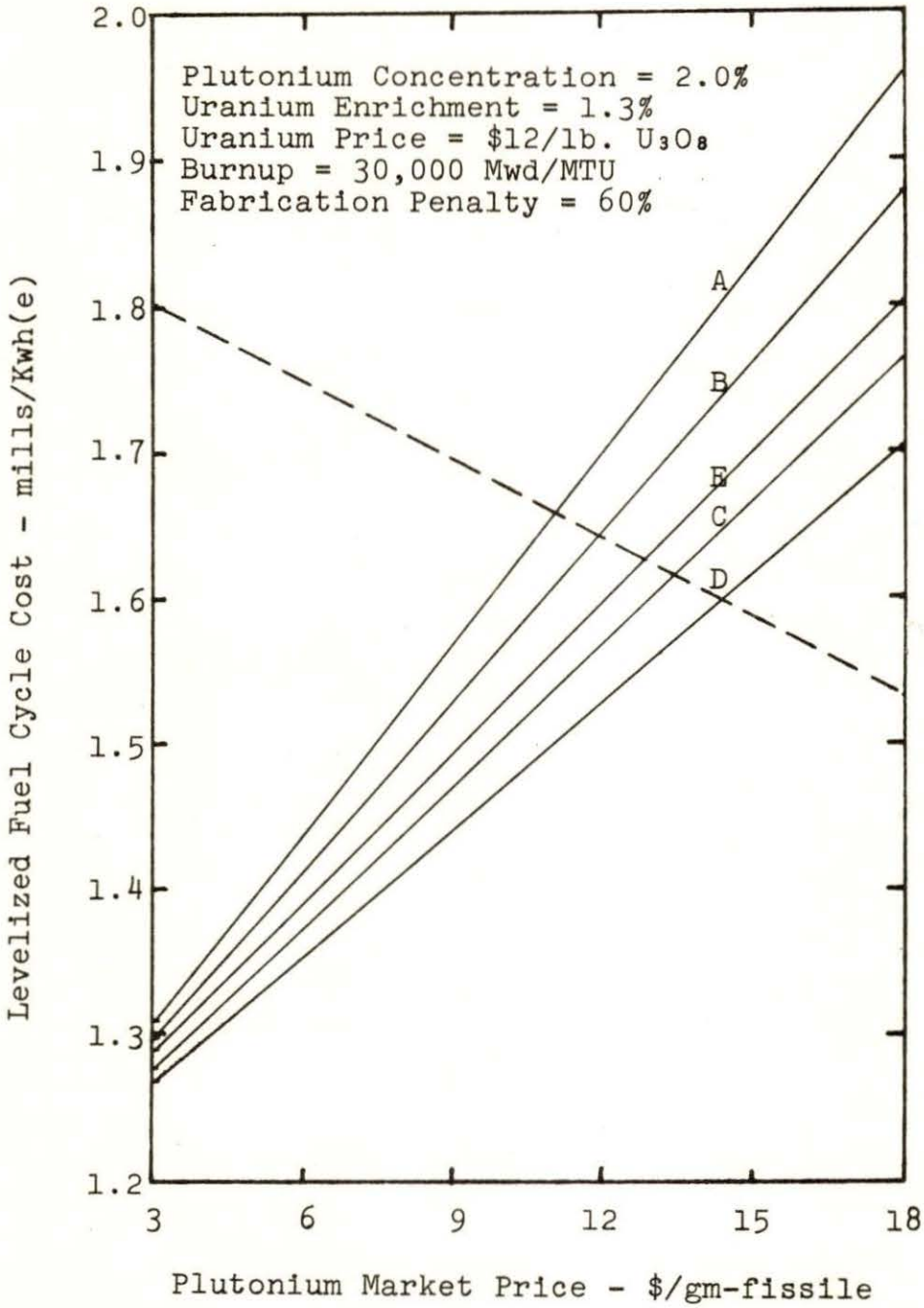

Figure 23. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

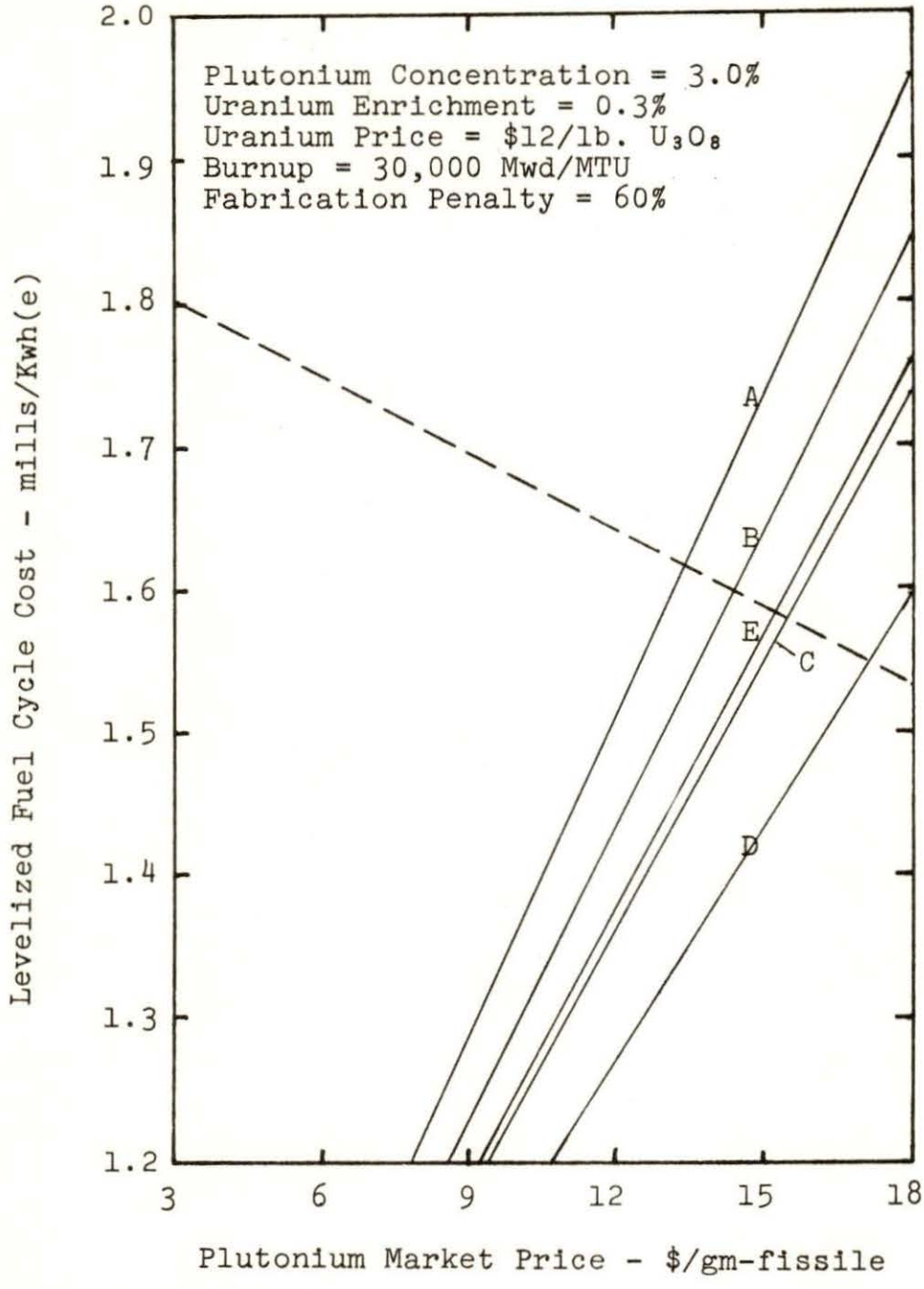

Figure 24. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

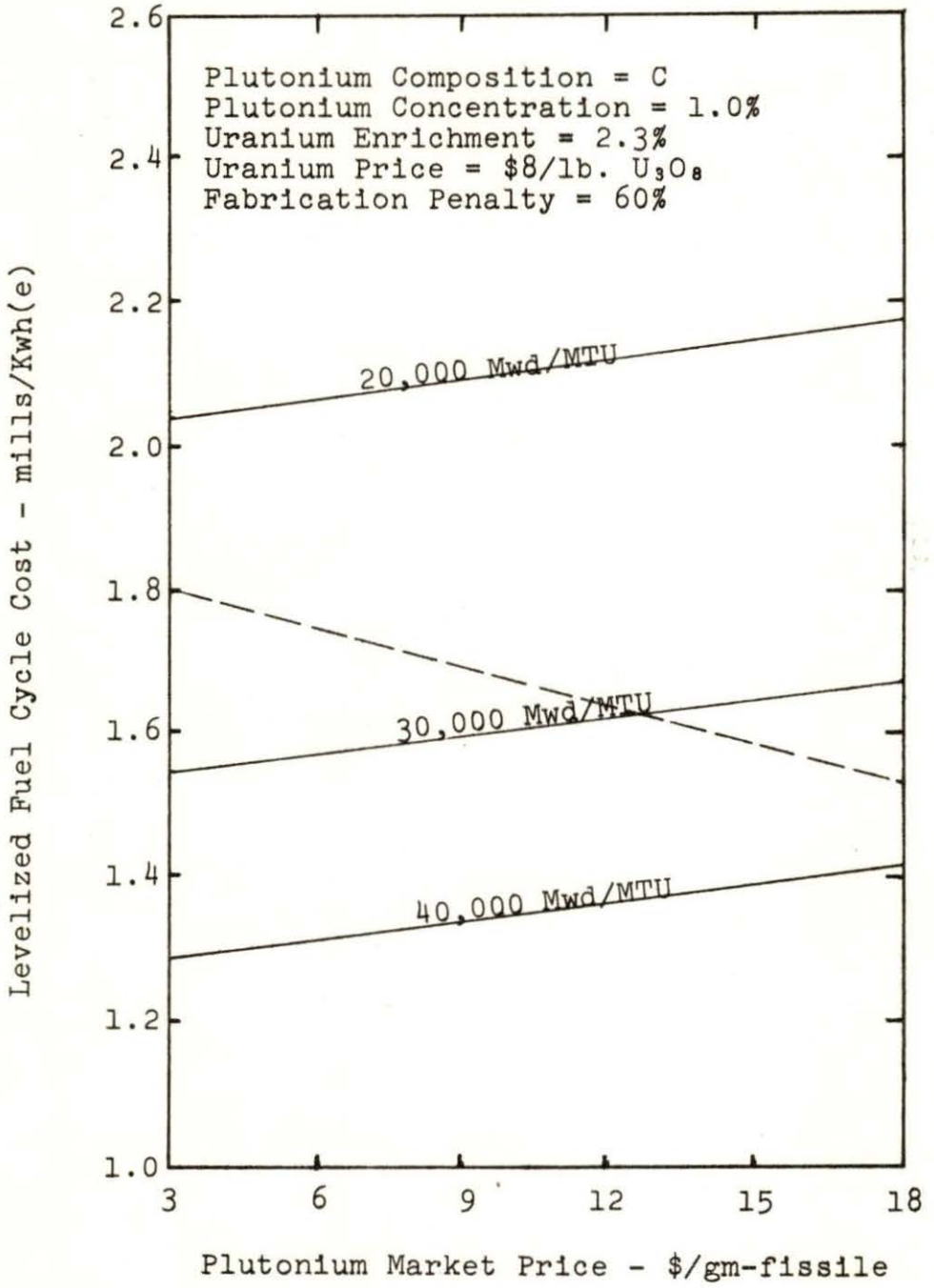

Figure 25. Levelized fuel cycle cost as a function of the plutonium market price and the burnup

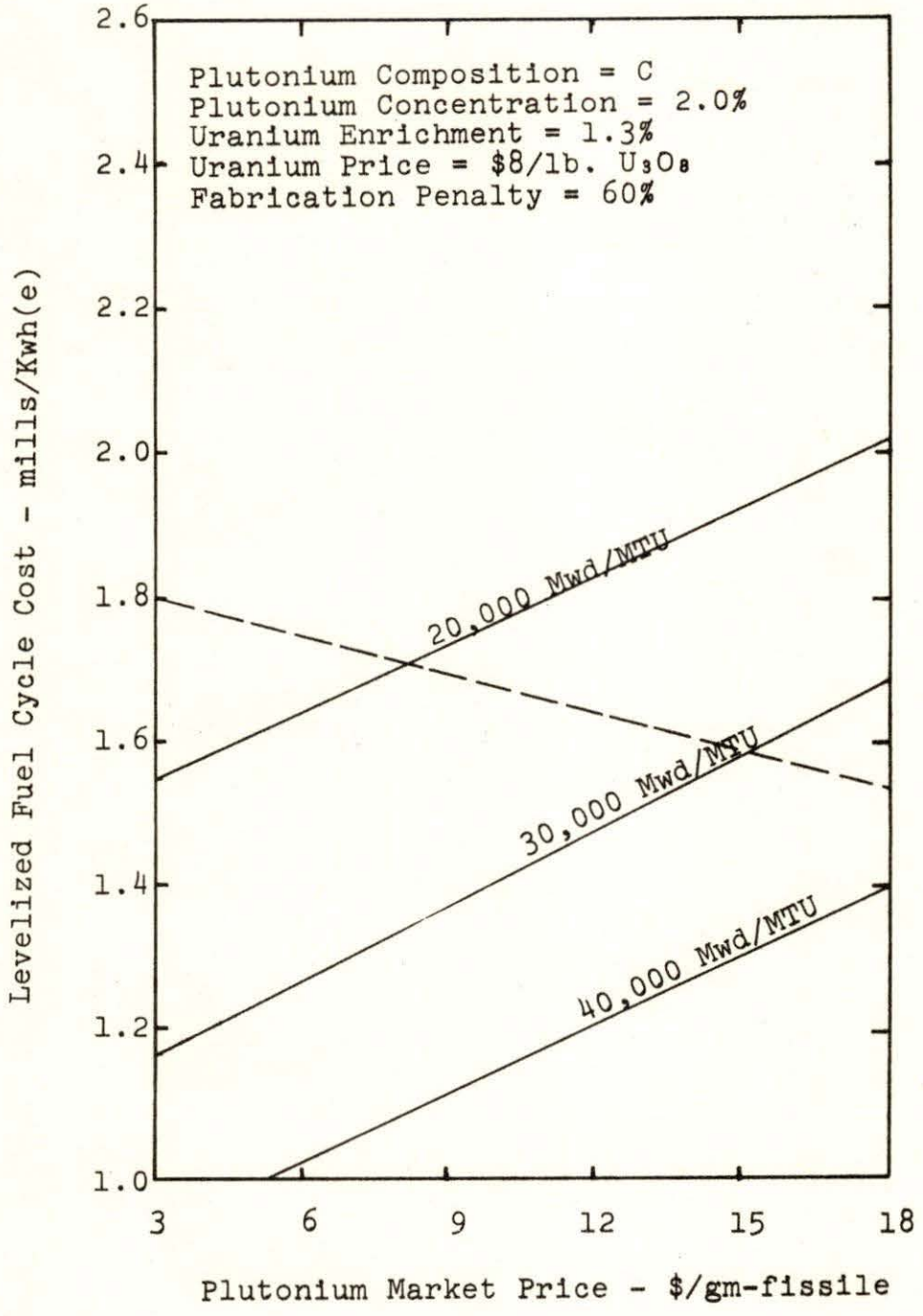

Figure 26. Levelized fuel cycle cost as a function of the market price of plutonium and the burnup

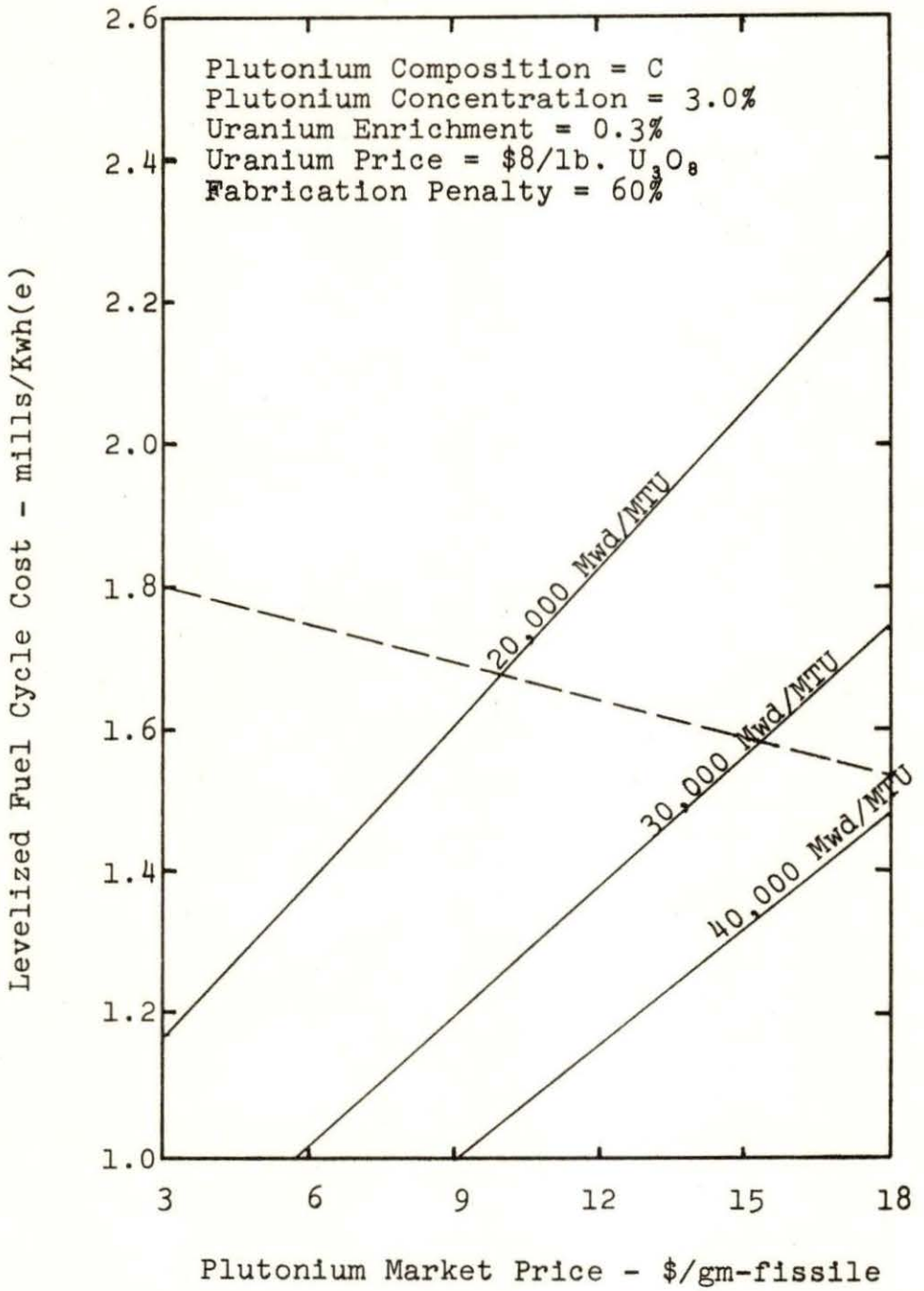

Figure 27. Levelized fuel cycle cost as a function of the market price of plutonium and the burnup

mixture to a value that gives the same reactivity at discharge for all fuel mixtures. The potentiometer on the analog computer representing the uranium enrichment was adjusted to give equal reactivities at discharge. In Figure 28 the reactivity of the fuel at 30,000 Mwd/MTU was equal to the reference uranium fuel reactivity at 30,000 Mwd/MTU. It should be noted that the uranium enrichment for fuel combinations B and C was less than 0.4 atom percent. The differences in the levelized fuel cycle costs were greater when a higher discharge reactivity was chosen, giving a higher initial uranium enrichment. Figure 29 illustrates the levelized fuel cycle costs for a higher reactivity at discharge. The results indicate that recycle fuels from discharged fuel of low burnup have the highest recycle value.

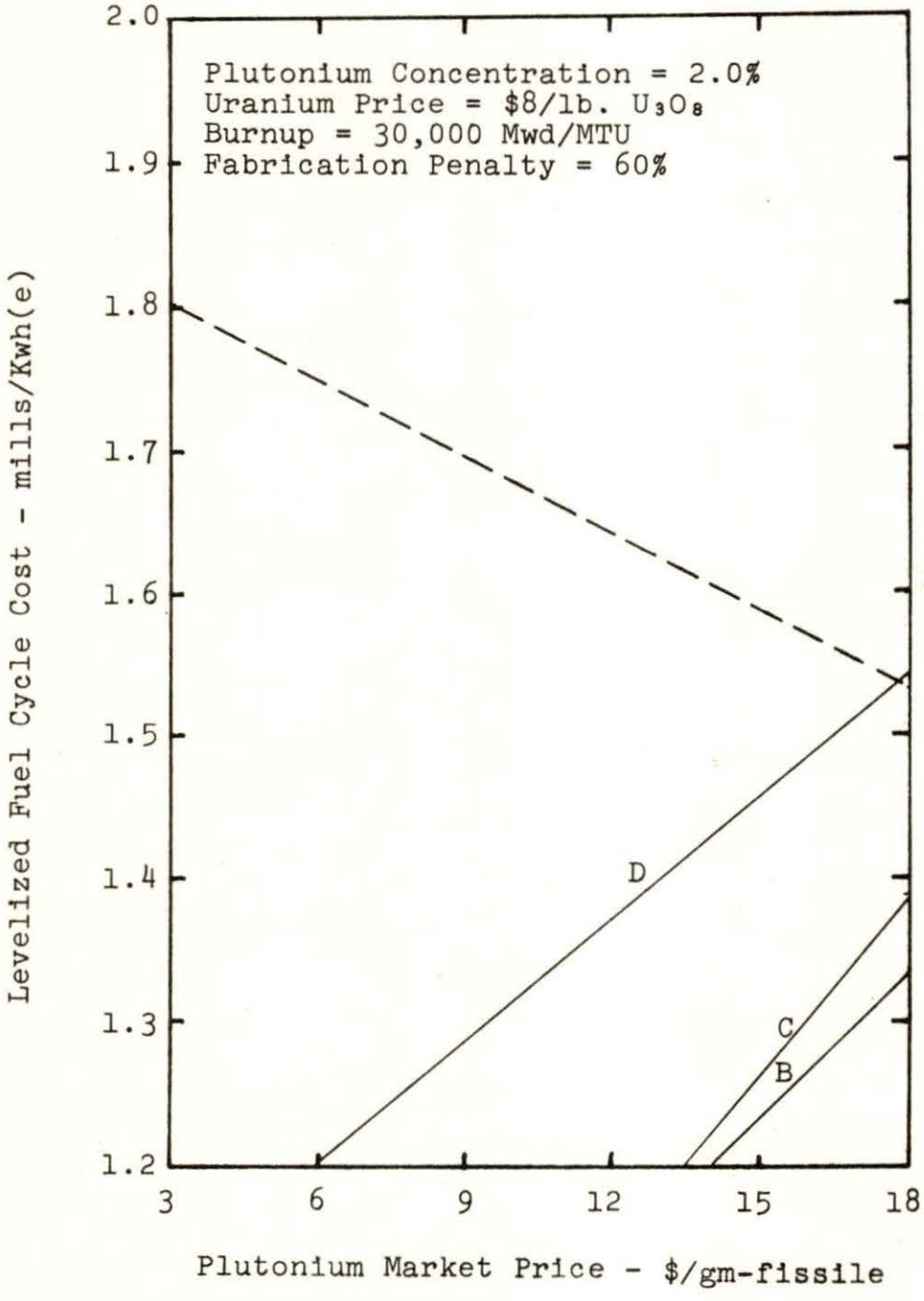

Figure 28. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

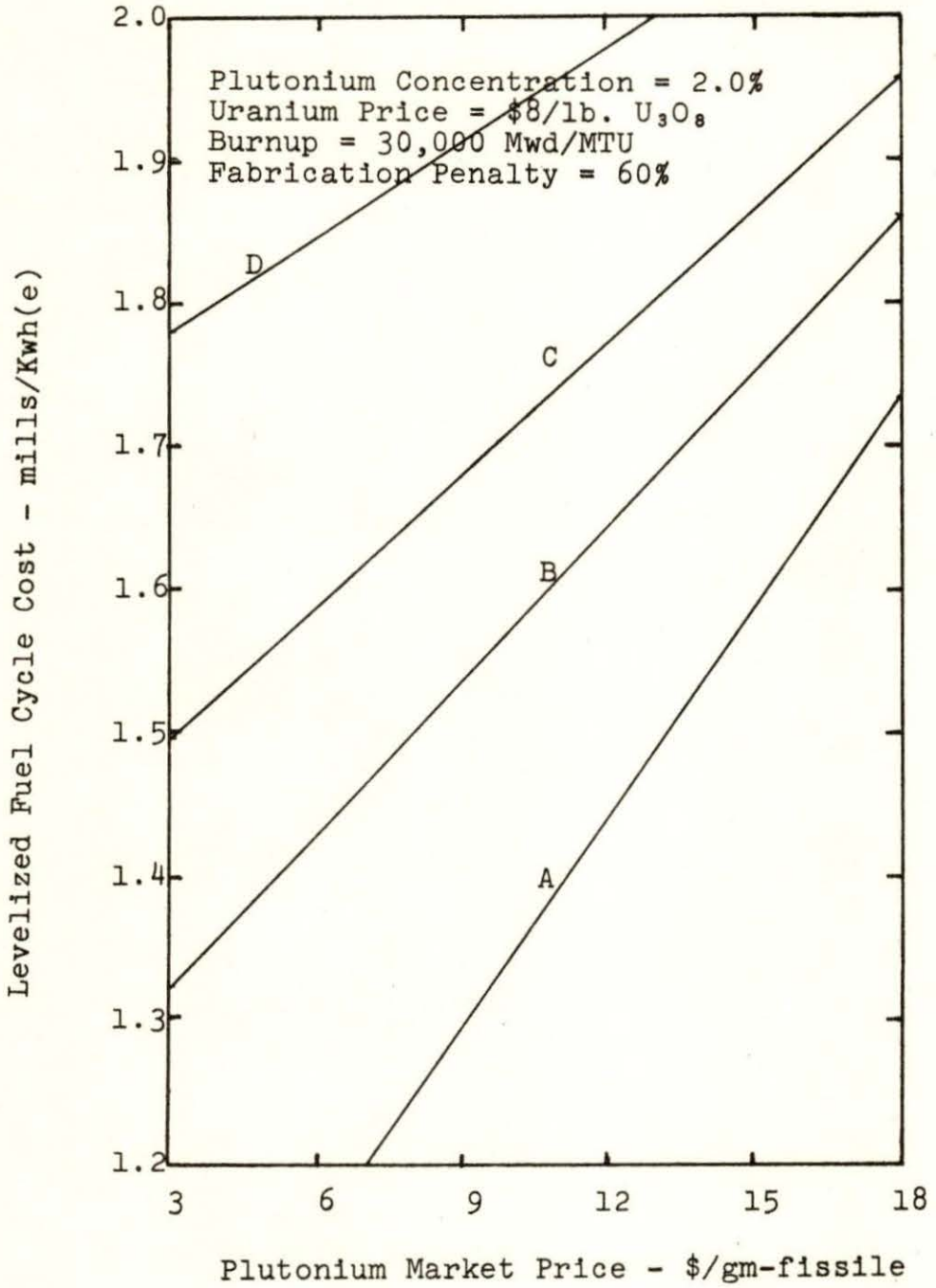

Figure 29. Levelized fuel cycle cost as a function of the plutonium market price and the plutonium composition

SUMMARY AND CONCLUSIONS

The levelized fuel cycle costs have been calculated for a number of different plutonium recycle fuel combinations, burnups, and fabrication penalties. A non-dimensional modified one group model was used to do the depletion calculations.

Some conclusions drawn from this study are as follows:

(1) As the market price of plutonium falls enough to encourage plutonium recycle, fuel mixtures of plutonium and uranium with a uranium-235 enrichment near that of naturally occurring uranium will be the most attractive fuel economically.

(2) If the market price of plutonium continues to fall, it may become more economical to use fuel mixtures consisting of plutonium and depleted uranium from diffusion plant tails.

(3) The fabrication penalty for plutonium-uranium fuel mixtures affects the economics of recycling plutonium by the greatest amount when small amounts of plutonium are recycled.

(4) Fabrication penalties in excess of 60 percent will prohibit the recycle of plutonium when it makes up 0.5 percent or less of the total fuel mixture.

(5) Although one isotopic mixture of plutonium may appear more economical than another for a given market price of plutonium, as the market price changes, the relative ec-

onomics of the two isotopic mixtures may also change. This may be true for fuel mixtures consisting of 1.5 percent or less plutonium in the fuel mixture. As the amount of plutonium in the fuel mixture approaches 2.0 percent this no longer appears to occur.

(6) The reactivities of arbitrarily contrived recycle fuel mixtures show large differences at discharge, and need to be considered to determine the relative values of isotopic mixtures of plutonium.

FUTURE STUDIES

During the course of this study possible ideas for future studies have developed. Since only a limited number of possible fuel mixtures were studied, other fuel mixtures which may be of interest could be studied. Another possibility might be to repeat the calculations here using a more accurate method for determining the flux and/or a multi-group calculation for determining the depletion and buildup of isotopes in the reactor.

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